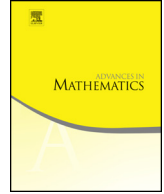




Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Chow ring of generically twisted varieties of complete flags [☆]



Nikita A. Karpenko

Mathematical & Statistical Sciences, University of Alberta, Edmonton, Canada

ARTICLE INFO

Article history:

Received 12 April 2016
Received in revised form 11 August 2016
Accepted 27 October 2016
Available online xxxx
Communicated by Aravind Asok

MSC:

20G15
14C25

Keywords:

Central simple algebras
Algebraic groups
Projective homogeneous varieties
Chow groups

ABSTRACT

Let G be a split simple affine algebraic group of type A or C over a field k , and let E be a standard generic G -torsor over a field extension of k . We compute the Chow ring of the *variety of Borel subgroups of G* (also called the *variety of complete flags of G*), twisted by E . In most cases, the answer contains a large finite torsion subgroup. The torsion-free cases have been treated in the predecessor *Chow ring of some generically twisted flag varieties* by the author.

© 2016 Elsevier Inc. All rights reserved.

Contents

1. Introduction	790
2. Specialization	791
3. Chow rings of Severi–Brauer varieties	793
4. Type A	803

[☆] This work has been supported by a Discovery Grant from the National Science and Engineering Board of Canada.

E-mail address: karpenko@ualberta.ca.

URL: <http://www.ualberta.ca/~karpenko>.

5. Type C	804
References	805

1. Introduction

Let G be a split semisimple affine algebraic group over a field k and let \mathcal{B} be its *variety of Borel subgroups* (also called the *variety of complete flags* of G). A rational point on \mathcal{B} is given by a Borel subgroup $B \subset G$. Picking up such a point, one establishes an isomorphism of \mathcal{B} with the quotient variety G/B . The Chow ring $\text{CH}\mathcal{B}$ of \mathcal{B} is well-studied and understood.

The situation changes dramatically when we twist \mathcal{B} by a G -torsor (i.e., a principle homogeneous space) E over a field extension $F \supset k$. The F -variety Y thus obtained is isomorphic to the quotient variety E/B . It is not reasonable to hope to understand $\text{CH}Y$ in general.

If the torsor E is split, the situation is easy because $Y \simeq \mathcal{B}_F$. Whatever viewpoint is applied to measure how far is a given torsor E from the split one, the *standard generic torsors* are always among the winners. A standard generic torsor E is, by definition, the generic fiber of the quotient map $\text{GL}_N \rightarrow \text{GL}_N/G$ for an imbedding of G into the general linear group GL_N for some $N \geq 1$. In particular, E is a G -torsor over the function field

$$F := k(\text{GL}_N/G) \supset k.$$

The Chow ring of the corresponding F -variety Y does not depend on the choice of the imbedding $G \hookrightarrow \text{GL}_N$ and is the main object of study in the paper.

A standard tool of studying $\text{CH}Y$ for an arbitrary smooth variety Y is the canonical epimorphism $\text{CH}Y \rightarrow GK(Y)$, where $K(Y)$ is the Grothendieck ring of Y endowed with the *topological filtration* (also called *geometrical filtration* as well as *filtration by codimension of support*) and $GK(Y)$ is the associated graded ring. The kernel of this epimorphism is contained in the torsion subgroup $\text{Tors CH}X$ of $\text{CH}X$ [6, Example 15.3.6] and is controlled by differentials of the Brown–Gersten–Quillen spectral sequence [16].

For our particular Y , the ring $\text{CH}Y$ is generated by its component $\text{CH}^1 Y$ [12, Example 2.4]. As a consequence, the topological filtration for our Y coincides with the gamma filtration [9, Remark 2.17]. The latter is the filtration generated by K -theoretical Chern classes, introduced by A. Grothendieck for arbitrary smooth varieties as an (at least theoretically) computable approximation of the topological filtration.

Here is the short version of the main result of the paper (see §4 and §5 for the proof):

Theorem 1.1. *Let G be a split simple affine algebraic group of type A or C and let Y be its variety of complete flags twisted by a standard generic torsor. Then the canonical epimorphism $\text{CH}Y \rightarrow GK(Y)$ is an isomorphism.*

Download English Version:

<https://daneshyari.com/en/article/4665028>

Download Persian Version:

<https://daneshyari.com/article/4665028>

[Daneshyari.com](https://daneshyari.com)