

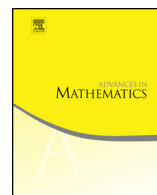


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Cluster structures on strata of flag varieties

B. Leclerc^{a,b,c,d,*}^a Normandie Univ., France^b UNICAEN, LMNO, F-14032 Caen, France^c CNRS UMR 6139, F-14032 Caen, France^d Institut Universitaire de France, France

ARTICLE INFO

Article history:

Received 25 February 2014

Accepted 31 December 2014

Available online 31 March 2016

In memory of Andrei Zelevinsky,
whose work has been a constant
source of inspiration.

Keywords:

Cluster algebra

Flag variety

Richardson variety

Preprojective algebra

ABSTRACT

We introduce some new Frobenius subcategories of the module category of a preprojective algebra of Dynkin type, and we show that they have a cluster structure in the sense of Buan–Iyama–Reiten–Scott. These categorical cluster structures yield cluster algebra structures in the coordinate rings of intersections of opposite Schubert cells.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

1.1. Let G be a simple and simply connected algebraic group over \mathbb{C} . We assume that G is simply-laced, that is, G is of type A , D , E in the Cartan–Killing classification. We fix a maximal torus H in G , a Borel subgroup B containing H , and we denote by B^- the Borel subgroup opposite to B with respect to H . Let $W = \text{Norm}_G(H)/H$ be the Weyl group, with length function $w \mapsto \ell(w)$ and longest element w_0 .

* Correspondence to: UNICAEN, LMNO, F-14032 Caen, France.

E-mail address: bernard.leclerc@unicaen.fr.

We consider the flag variety $X = B^- \backslash G$, and we denote by $\pi : G \rightarrow X$ the natural projection $\pi(g) := B^-g$. The Bruhat decomposition

$$G = \bigsqcup_{w \in W} B^-wB^-$$

projects to the Schubert decomposition

$$X = \bigsqcup_{w \in W} C_w, \tag{1}$$

where $C_w = \pi(B^-wB^-)$ is the *Schubert cell* attached to w , isomorphic to $\mathbb{C}^{\ell(w)}$. We may also consider the Birkhoff decomposition

$$G = \bigsqcup_{w \in W} B^-wB,$$

which projects to the *opposite* Schubert decomposition

$$X = \bigsqcup_{w \in W} C^w, \tag{2}$$

where $C^w = \pi(B^-wB)$ is the *opposite Schubert cell* attached to w , isomorphic to $\mathbb{C}^{\ell(w_0) - \ell(w)}$. The intersection

$$\mathcal{R}_{v,w} := C^v \cap C_w \tag{3}$$

has been considered by Kazhdan and Lusztig [34] in relation with the cohomological interpretation of the Kazhdan–Lusztig polynomials. One shows [34,13] that $\mathcal{R}_{v,w}$ is non-empty if and only if $v \leq w$ in the Bruhat order of W , and it is a smooth irreducible locally closed subset of C_w of dimension $\ell(w) - \ell(v)$. More recently, $\mathcal{R}_{v,w}$ has sometimes been called an open Richardson variety [36], because its closure in X is known as a Richardson variety [44].

Intersecting the decompositions (1) and (2) of X , we thus get a finer stratification

$$X = \bigsqcup_{v \leq w} \mathcal{R}_{v,w}. \tag{4}$$

However, in contrast with (1) or (2), the strata $\mathcal{R}_{v,w}$ of (4) are not isomorphic to affine spaces.

1.2. Let I be the vertex set of the Dynkin diagram of G . We denote by $x_i(t)$ ($i \in I$, $t \in \mathbb{C}$) (resp. $y_i(t)$ ($i \in I$, $t \in \mathbb{C}$)) the one-parameter subgroups of B (resp. B^-) attached to the simple roots. For $K \subset I$, let B_K^- be the standard parabolic subgroup of G generated by B^- and the $x_i(t)$ with $i \in K$. We denote by $X_K = B_K^- \backslash G$ the corresponding partial flag variety. Let $\pi_K : G \rightarrow X_K$ and $\pi^K : X \rightarrow X_K$ be the natural projections, so that we have $\pi_K = \pi^K \circ \pi$. Let W_K be the parabolic subgroup of W corresponding to K with longest element w_K , and let W^K be the subset of W_K consisting of the minimal

Download English Version:

<https://daneshyari.com/en/article/4665077>

Download Persian Version:

<https://daneshyari.com/article/4665077>

[Daneshyari.com](https://daneshyari.com)