

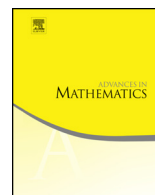


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Integrable cluster dynamics of directed networks and pentagram maps



Michael Gekhtman^{a,*}, Michael Shapiro^b, Serge Tabachnikov^{c,d},
Alek Vainshtein^e

^a Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556, USA

^b Department of Mathematics, Michigan State University, East Lansing, MI 48823, USA

^c Department of Mathematics, Pennsylvania State University, University Park, PA 16802, USA

^d ICERM, Brown University, Providence, RI 02903, USA

^e Department of Mathematics and Department of Computer Science, University of Haifa, Haifa, Mount Carmel 31905, Israel

ARTICLE INFO

Article history:

Received 9 June 2014

Accepted 3 December 2014

Available online 30 March 2016

To the memory of Andrei Zelevinsky

Keywords:

Generalized pentagram maps

Liouville integrability

Cluster dynamics

Mutations

Networks on surfaces

ABSTRACT

The pentagram map was introduced by R. Schwartz more than 20 years ago. In 2009, V. Ovsienko, R. Schwartz and S. Tabachnikov established Liouville complete integrability of this discrete dynamical system. In 2011, M. Glick interpreted the pentagram map as a sequence of cluster transformations associated with a special quiver. Using compatibility of Poisson and cluster structures and Poisson geometry of directed networks on surfaces, we generalize Glick's construction to include the pentagram map into a family of discrete integrable maps and we give these maps geometric interpretations. The appendix relates the simplest of these discrete maps to the Toda lattice and its tri-Hamiltonian structure.

© 2016 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: mgekhtma@nd.edu (M. Gekhtman), mshapiro@math.msu.edu (M. Shapiro), tabachni@math.psu.edu (S. Tabachnikov), alek@cs.haifa.ac.il (A. Vainshtein).

Contents

1.	Introduction	391
2.	Generalized Glick’s quivers and the (\mathbf{p}, \mathbf{q}) -dynamics	394
3.	Weighted directed networks and the (\mathbf{x}, \mathbf{y}) -dynamics	397
3.1.	Weighted directed networks on surfaces	397
3.2.	The (\mathbf{x}, \mathbf{y}) -dynamics	400
4.	Poisson structure and complete integrability	407
4.1.	Cuts, rims, and conjugate networks	408
4.2.	Poisson structure	413
4.3.	Conserved quantities	415
4.4.	Lax representations	418
4.5.	Complete integrability	421
4.6.	Spectral curve	423
5.	Geometric interpretation	424
5.1.	The case $k \geq 3$	424
5.1.1.	Corrugated polygons and generalized higher pentagram maps	424
5.1.2.	Coordinates in the space of corrugated polygons	428
5.1.3.	Higher pentagram maps on plane polygons	433
5.2.	The case $k = 2$: leapfrog map and circle pattern	435
5.2.1.	Space of pairs of twisted n -gons in \mathbf{RP}^1	435
5.2.2.	Leapfrog transformation	436
5.2.3.	Lagrangian formulation of leapfrog transformation	438
5.2.4.	Circle pattern	440
	Acknowledgments	441
	Appendix A. Leapfrog map and Toda lattice (by Anton Izosimov)	442
A.1.	Continuous limit of the leapfrog map and the Toda lattice	442
A.2.	Hamiltonian structure of the leapfrog flow and the cubic Toda bracket	444
A.3.	Tri-Hamiltonian structure of the map T_2	446
	References	449

1. Introduction

The pentagram map was introduced by R. Schwartz more than 20 years ago [36]. The map acts on plane polygons by drawing the “short” diagonals that connect second-nearest vertices of a polygon and forming a new polygon, whose vertices are their consecutive intersection points, see Fig. 1. The pentagram map commutes with projective transformations, and therefore acts on the projective equivalence classes of polygons in the projective plane.

In fact, the pentagram map acts on a larger class of *twisted polygons*. A twisted n -gon is an infinite sequence of points $V_i \in \mathbf{RP}^2$ such that $V_{i+n} = M(V_i)$ for all $i \in \mathbf{Z}$ and a fixed projective transformation M , called the monodromy. The projective group $\text{PGL}(3, \mathbf{R})$ naturally acts on twisted polygons. A polygon is closed if the monodromy is the identity.

Denote by \mathcal{P}_n the moduli space of projective equivalence classes of twisted n -gons, and by \mathcal{C}_n its subspace consisting of closed polygons. Then \mathcal{P}_n and \mathcal{C}_n are varieties of dimensions $2n$ and $2n - 8$, respectively. Denote by $T : \mathcal{P}_n \rightarrow \mathcal{P}_n$ the pentagram map (the i th vertex of the image is the intersection of diagonals (V_i, V_{i+2}) and (V_{i+1}, V_{i+3})).

Download English Version:

<https://daneshyari.com/en/article/4665082>

Download Persian Version:

<https://daneshyari.com/article/4665082>

[Daneshyari.com](https://daneshyari.com)