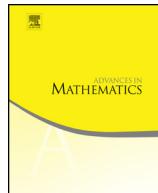




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Approximation on Nash sets with monomial singularities



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ABSTRACT

This paper is devoted to the approximation of differentiable semialgebraic functions by Nash functions. Approximation by Nash functions is known for semialgebraic functions defined on an affine Nash manifold M , and here we extend it to functions defined on *Nash sets* $X \subset M$ whose singularities are monomial. To that end we discuss first finiteness and weak normality for such sets X . Namely, we prove that (i) X is the union of finitely many open subsets, each Nash diffeomorphic to a finite union of coordinate linear varieties of an affine space, and (ii) every function on X which is Nash on every irreducible component of X extends to a Nash function on M . Then we can obtain approximation for semialgebraic functions and even for certain semialgebraic maps on Nash sets with monomial singularities. As a nice consequence we show that m -dimensional affine Nash manifolds with divisorial corners which are class k semialgebraically diffeomorphic, for $k > m^2$, are also Nash diffeomorphic.

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1. Introduction

The importance of approximation in the Nash setting arises from the fact that this category is highly rigid to work with; for instance, it does not admit partitions of unity, a usual and fruitful tool when available. On the other hand, there exist finite differentiable semialgebraic partitions of unity and therefore approximation becomes interesting as a bridge between the differentiable semialgebraic category and the Nash one. There are already relevant results concerning absolute approximation in the Nash setting (e.g. Effroymson's approximation theorem [9, §1]). An even more powerful tool is relative approximation that allows approximation having a stronger control over certain subsets. In the '80s Shiota developed a thorough study of Nash manifolds and Nash sets (see [15] for the full collected work); among other things, he devised approximation on an affine Nash manifold *relative to a Nash submanifold*. Our purpose is to generalize this type of results developing Nash approximation on an affine Nash manifold relative to a Nash subset; of course, as one can expect we need some conditions concerning the singularities on the Nash subset and we will focus on those whose singularities are of *monomial type*. This will require a careful preliminary study of this type of singularities in the Nash context. As an interesting application of our results, we prove that the Nash classification of affine m -dimensional Nash manifolds with (divisorial) corners is equivalent to the \mathcal{C}^k semialgebraic classification for $k > m^2$.

Recall that a set $X \subset \mathbb{R}^n$ is *semialgebraic* if it is a Boolean combination of sets defined by polynomial equations and inequalities. A semialgebraic set $M \subset \mathbb{R}^n$ is called an (*affine*) *Nash manifold* if it is moreover a smooth submanifold of (an open subset of) \mathbb{R}^n ; as in this paper all manifolds are affine we often drop the word affine. A function

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