Arab Journal of Mathematical Sciences

Arab J Math Sci 22 (2016) 132–137

On the existence of positive solutions for an ecological model with indefinite weight

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Received 10 September 2014; received in revised form 1 December 2014; accepted 9 December 2014
Available online 12 February 2015

Abstract. This study concerns the existence of positive solutions for the following nonlinear boundary value problem:

$$\begin{cases} -\Delta u = am(x)u - bu^2 - c\dfrac{u^p}{u^p+1} - K & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Delta u = \operatorname{div}(\nabla u)$ is the Laplacian of u, while a, b, c, p, K are positive constants with $p \geq 2$ and Ω is a bounded smooth domain of $\mathbb{R}^N$ with $\partial\Omega$ in C^2. The weight function m satisfies $m \in C(\Omega)$ and $m(x) \geq m_0 > 0$ for $x \in \Omega$, also $\|m\|_\infty = l < \infty$. We prove the existence of positive solutions under certain conditions.

Keywords: Ecological systems; Indefinite weight; Grazing and constant yield harvesting; Sub–super solution method

2010 Mathematics Subject Classification: 35J65; 35J25

1. Introduction

In this note, we mainly consider the following reaction–diffusion equation:

$$\begin{cases} -\Delta u = am(x)u - bu^2 - c\dfrac{u^p}{u^p+1} - K & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

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Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.ajmsc.2014.12.002

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where $\Delta u = \mathrm{div}(\nabla u)$ is the Laplacian of u, while a, b, c, p, K are positive constants with $p \geq 2$ and Ω is a bounded smooth domain of $\mathbb{R}^N$ with $\partial\Omega$ in C^2. The weight function m satisfies $m \in C(\Omega)$ and $m(x) \geq m_0 > 0$ for $x \in \Omega$, also $\|m\|_\infty = l < \infty$. We denote by λ_1 the first eigenvalue of

$$\begin{cases} -\Delta\phi = \lambda m(x)\phi & x \in \Omega, \\ \phi = 0 & x \in \partial\Omega, \end{cases} \tag{1.2}$$

with positive principal eigenfunction ϕ_1 satisfying $\|\phi_1\|_\infty = 1$ (see [4]).

Here u is the population density and $am(x)u - bu^2$ represents the logistics growth. This model describes grazing of a fixed number of grazers on a logistically growing species (see [6,8]). The herbivore density is assumed to be a constant which is a valid assumption for managed grazing systems and the rate of grazing is given by $\frac{cu^p}{1+u^p}$. At high levels of vegetation density this term saturates to c as the grazing population is a constant. This model has also been applied to describe the dynamics of fish populations (see [6,12]). The diffusive logistic equation with constant yield harvesting, in the absence of grazing was studied in [9]. Recently, in the case when $m(x) = 1$ problem (1.1) has been studied by R. Shivaji et al. (see [2]).

The purpose of this paper is to improve the result of [2] with weight m. We shall establish our abstract existence result via the method of sub–super solutions. The concepts of sub–super solution were introduced by Nagumo [7] in 1937 who proved, using also the shooting method, the existence of at least one solution for a class of nonlinear Sturm–Liouville problems. In fact, the premises of the sub–super solutions method can be traced back to Picard. He applied, in the early 1880s, the method of successive approximations to prove the existence of solutions for nonlinear elliptic equations that are suitable perturbations of uniquely solvable linear problems. This is the starting point of the use of sub–super solutions in connection with monotone methods. Picard's techniques were applied later by Poincaré [10] in connection with problems arising in astrophysics. We refer the reader to [11].

Definition 1.1. We say that ψ (*resp.* z) in $C^2(\Omega) \cap C(\overline{\Omega})$ is a *subsolution* (*resp. super solution*) of (1.1), if ψ (*resp.* z) satisfies

$$\begin{cases} -\Delta\psi \leq am(x)\psi - b\psi^2 - c\dfrac{\psi^p}{\psi^p + 1} - K & \text{in } \Omega, \\ \psi \geq 0 & \text{in } \Omega, \\ \psi = 0 & \text{on } \partial\Omega \end{cases} \tag{1.3}$$

$$\left(\textit{resp.} \begin{cases} -\Delta z \geq am(x)z - bz^2 - c\dfrac{z^p}{z^p + 1} - K & \text{in } \Omega, \\ z \geq 0 & \text{in } \Omega, \\ z = 0 & \text{on } \partial\Omega \end{cases} \right). \tag{1.4}$$

Then the following lemma holds (see [1]).

Lemma 1.2 (*See [1]*)**.** *If there exist sub–super solutions ψ and z respectively, such that $\psi \leq z$ on Ω, Then* (1.1) *has a positive solution u such that $\psi \leq u \leq z$ in Ω.*

Proposition 1.3. *If $a \leq \lambda_1$ then* (1.1) *has no positive solution.*

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