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A non-linear constitutive model for describing the mechanical behaviour of frozen ground and permafrost

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ABSTRACT

The mechanical behaviour of frozen ground and permafrost is changing under the increasing variation of environmental and anthropogenic boundary conditions. This phenomenon affects many civil structures and infrastructures built in Polar and Alpine areas. Mathematical formulations able to capture the mechanical behaviour of frozen ground and permafrost with adherence to reality and a limited employment of technical resources and time appear crucial for the engineering design and retrofit of these structures. To address this challenge, this study presents a relatively simple elasto-plastic constitutive model for capturing the non-linear mechanical behaviour of frozen silt. The model is based on associated flow rules. It employs an elliptical yield surface and a parabolic yield surface for describing the volumetric mechanisms that characterise the modelled material, together with a parabolic yield surface for describing the deviatoric mechanism. Comparisons with experimental triaxial test results available in the literature highlight the suitability of the model to capture the non-linear mechanical response of frozen silt subjected to both low and high confining pressures. This result, together with the doable implicit consideration in the model of the effects induced by environmental boundary conditions such as temperature on the mechanical behaviour of the material, makes this tool attractive for simplified yet thorough analyses of frozen ground and permafrost-related problems.

1. Introduction

Frozen ground and permafrost in Polar (high latitudes, i.e., regions close to the North and South poles) and Alpine (high elevations in lower latitudes, i.e., 2200–3500 m a.s.l.) areas are more often negatively affected by a degradation of mechanical properties. Severe damages to buildings and infrastructures (Haeberli, 1992; Phillips et al., 2007; Phillips and Margreth, 2008; Arenson et al., 2009, Romanovsky and Osterkamp, 2001; Clarke et al., 2008; Instanes, 2003; Streletskiy et al., 2014; Duvillard et al., 2015) as well as phenomena of slope and flat ground instability (Davies et al., 2001; Gruber et al., 2004; Gruber and Haeberli, 2007; Haeberli, 1985; Huggel, 2009; Arenson and Springman, 2000; Nelson et al., 2003; Romanovsky et al., 2007; Arnold et al., 2005; Pei et al., 2014; Shan et al., 2014; Stoffel et al., 2014; Deline et al., 2015; Haeberli et al., 2016) are increasingly observed in such areas. These phenomena are generally induced by the combined action of climate change, construction activity and existing structures, which involves variations of the temperature and stress fields characterising the substrate and results in detrimental changes of the mechanical behaviour of the ground. In fact, besides the strong dependence of the strength and stiffness of frozen ground and permafrost on intrinsic material properties such as moisture content, air bubbles, salts, organic matter and grain sizes (Parameswaran and Jones, 1998; Fukuo, 1966a,b; Anderson and Morgenstern, 1973; Sayles, 1988; Ladanyi, 1981, 1985; Andersland and Ladanyi, 2004; Arenson et al., 2007, 2014; Springman and Arenson, 2008; among others), and on different stress histories and strain rates/paths (Andersland and Ainouri, 1970; Baker, 1979; Fish, 1981; Zhu and Carbee, 1987; Wu and Ma, 1994; Guryanov and Ma, 1995; Arenson et al., 2004; Lai et al., 2014, among others), confining pressures and thermal loads are crucial for the variation of the mechanical behaviour of these materials (Chamberlain et al., 1972; Alkire and Andersland, 1973; Baker et al., 1982; Ting et al., 1983; Andersen et al., 1995; Cuccovillo and Coop, 1997; Da Re et al., 2003; Arenson and Springman, 2005a,b; Yang et al., 2010; Yamamoto and Springman, 2012, among others). The strength of frozen ground and permafrost generally increases with greater applied confining pressures, but may abruptly decrease under high confining pressures because of pressure melting and

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crushing phenomena. These phenomena may be enhanced by temperature variations, especially when changes of the phases that characterise the matrix constituents may occur.

In an attempt to model the influence of the aforementioned variables on the mechanical behaviour of frozen ground and permafrost, mathematical formulations such as constitutive models and strength criteria are increasingly formulated and exploited, as they are the basis to describe the response of bodies under loading. Currently, besides a series of formulations that have been proposed for describing the mechanical behaviour of frozen ground and permafrost subjected to uniaxial loads (Zhu and Carbee, 1987; Zhu et al., 1992), long-term loads (Goughnour and Andersland, 1968; Ladanyi, 1972; Sayles, 1973; Fish, 1983, 1984; Zhu and Carbee, 1983; Gardner et al., 1984; Cai et al., 1990; Domaschuk et al., 1991; Wijeweera and Joshi, 1991; Miao et al., 1995; He et al., 2000; Yang et al., 2004; Wang et al., 2006, 2014), and evolutionary thermal fields (Arenson and Springman, 2005b; Li et al., 2009; Yang et al., 2010) with the consideration of a number of associated problems (Nishimura et al., 2009; Thomas et al., 2010; Zhang and Michalowski, 2015; Zhang et al., 2016), models for capturing the effects of confining loads have been presented (Ma et al., 1995; Lai et al., 2009, 2010, 2013, 2016; Yang et al., 2010; Li et al., 2010; Liao, 2016). However, when attempting to capture the highly non-linear mechanical behaviour of frozen ground and permafrost under variations of confining pressure, there is the main issue of needing to use complex mathematical formulations that rely on many material parameters. This need often results in challenging and time-consuming analyses of real engineering problems, with a number of technical and economical shortcomings for the engineering practice. The proposition of mathematical formulations based on fewer material parameters and able to capture with suitable adherence reality seems important in this scope.

Looking at such challenge, this study presents an additional constitutive model for performing preliminary analyses of the mechanical behaviour of frozen ground and permafrost. In particular, this paper proposes a non-linear elasto-plastic constitutive model for describing the mechanical response of frozen silt subjected to different magnitudes of confining pressure, based on the groundwork performed by Lai et al. (2010). The goal of this work is to develop a mathematical tool that may be expediently and effectively applied for performing simplified yet thorough numerical analyses of frozen ground and permafrost-related problems. In the following, the elasto-plastic constitutive model is first formulated. The differences from the model presented by Lai et al. (2010) are discussed along the lines. Forethoughts that may allow to implicitly account through the proposed model for the effects of environmental boundary conditions such as temperature on the mechanical behaviour of the modelled material are also presented. Then, the constitutive model is applied for simulating the mechanical behaviour of frozen silt tested in triaxial conditions (Lai et al., 2010) under both low and high confining pressures, and is afterwards validated. Finally, comments on the capability of the constitutive model for capturing the non-linear mechanical behaviour of frozen ground and permafrost are summarised, and related concluding remarks are outlined.

2. Elasto-plastic constitutive modelling of frozen silt

2.1. Hypotheses and components of the model

The following constitutive model is referred to an isotropic and homogeneous material that is characterised by an elasto-plastic behaviour. The formulation of the model comprises the definition of four key components: (*i*) the relations governing the reversible behaviour of the material (elastic deformation); (*ii*) the criteria (yield functions) characterising the limit for which an irreversible behaviour may occur (plastic deformation); (*iii*) the relations (plastic potential functions) characterising the irreversible behaviour of the material, including the definition of the mechanisms governing the development of plastic deformation and the direction of its evolution; and (*iv*) the expressions (hardening rules) describing the magnitude of plastic deformation. Aspects (*i*) and (*ii*–*iv*) characterise the elastic and plastic constitutive description of any modelled material, respectively.

According to the theory of plasticity, the total strain tensor

$$\epsilon = \epsilon^e + \epsilon^p \tag{1}$$

can be decomposed into a sum of two symmetric tensors, i.e., the elastic strain tensor, ϵ^e, and the plastic strain tensor, ϵ^p. It is assumed that the modelled material exhibits plastic flow, but suffers no damage. Therefore, the elastic stiffness of the material remains unchanged. The stress-strain relation is in this case:

$$\boldsymbol{\sigma} = \boldsymbol{C}^e : \epsilon^e = \boldsymbol{C}^e : (\epsilon - \epsilon^p) \tag{2}$$

where $\boldsymbol{\sigma}$ is the total stress tensor and $\boldsymbol{C}^e$ is the elasticity matrix.

To determine the complete stress-strain relation, the elastic and plastic strain tensors need to be determined. Attention is given to these aspects in the sections that follow.

2.2. Elastic constitutive description

In this study, the elastic part of the strain is defined through the relation between the shear modulus, G, and the bulk modulus, K, which reads

$$K = \frac{2(1+\nu)}{3(1-2\nu)}G \tag{3}$$

where ν is the Poisson's ratio, which can be determined through the formula proposed by Bardet (1997), for example. As it has been remarked in the literature (Andersland and Ladanyi, 2004; Lai et al., 2009, 2010; Yang et al., 2010), the shear modulus depends on factors such as atmospheric pressure, p_a, and confining pressure, σ_3. The shear modulus generally increases at a first extent up to a maximum value with increasing confining pressure and then decreases for a further increase of confining pressure. Looking at a thorough description of this phenomenon, the empirical parabolic relation proposed by Lai et al. (2010) is employed in the following:

$$G = \left[a_g \left(\frac{\sigma_3}{p_a} \right)^2 + b_g \left(\frac{\sigma_3}{p_a} \right) + c_g \right] p_a \tag{4}$$

where a_g, b_g and c_g are material constants.

Knowledge of the elastic parameters presented above allows calculating the elastic components of deformation (i.e., volumetric and deviatoric) in a conventional way. Decomposition of the total strain in the elastic and plastic components (Eq. (1)) allows next to face the problem of determining the plastic component of deformation.

2.3. Plastic constitutive description

2.3.1. Yield functions

According to Yao et al. (2004), the yield surfaces of frozen soils and rocks are as follows: (*i*) smooth and convex in π-plane, i.e., the two-dimensional setting passing through the origin of the principal stress space where $\sigma_1 + \sigma_2 + \sigma_3 = const$, and p-q-plane, i.e., the two-dimensional setting where the mean stress p is paired with the generalised deviatoric stress q (Roscoe and Burland, 1968; Muir Wood, 1990); (*ii*) characterised by a critical state line that passes through the isotropic tensile point ($r_1 = r_2 = r_3 = f_{ttt}$) instead of the origin, differently from most geomaterials;