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Construction of nonsingular formulae of variance and covariance function of disturbing gravity gradient tensors

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Abstract: When the computational point is approaching the poles, the variance and covariance formulae of the disturbing gravity gradient tensors tend to be infinite, and this is a singular problem. In order to solve the problem, the authors deduced the practical non-singular computational formulae of the first- and second-order derivatives of the Legendre functions and two kinds of spherical harmonic functions, and then constructed the nonsingular formulae of variance and covariance function of disturbing gravity gradient tensors.

Key words: nonsingular; gravity field model; satellite gravity gradient; variance; covariance

1 Introduction

The least squares collocation (LSC) method which is widely used, is one of the most important research results in the approximation of the Earth's gravity field model (EGM) in recent era. Especially with the implementation and development of GOCE mission, this method attracts more and more attentions and its theory is further developed and perfected as it is one of the methods for determining the global and local EGM.

When using LSC to determine the EGM from GOCE satellite gravity gradient (SGG) data, the variance and covariance formulae of the disturbing gravity tensors and the gravity potential coefficients must be computed, however, when the computational point is approaching the poles, the variance and covariance formulae of the disturbing gravity tensors will tend to be infinite, and this is a singular problem. Along with the development of science and technology, especially the implementation of the satellite gravimetry, it is more and more important to deal with the singular problem.

Recently, domestic and foreign scholars did some valuable studies on the solution of the singular problem, based on their researches, some detailed problems which have not yet been solved completely are studied further in this paper. Firstly, the traditional computational formulae of variance and covariance function of disturbing gravity gradient tensors are constructed, then, the nonsingular computational formulae of the first- and second-order derivatives of the Legendre functions and two kinds of spherical harmonic functions are deduced, finally, the nonsingular formulae of variance and covariance function of disturbing gravity gradient tensors are constructed.

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2 Covariance function of disturbing potential

The space covariance function of disturbing potential can be expressed as follows[1-4]:

$$K(P,Q)=\sum_{n=2}^{\infty}\left(\frac{R^2}{rr'}\right)^{n+1}K_nP_n(\cos\psi) \quad (1)$$

where P and Q are two points in space, R is the Earth's average radius, r and r' are the geocentric radius vector of P and Q, respectively, $P_n(\cos\psi)$ is the Legendre polynomial, ψ is the geocentric angle distance between P and Q, and its expression is

$$\cos\psi=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\lambda-\lambda') \quad (2)$$

where (θ,λ) are the geocentric colatitude and geocentric longitude of P, and (θ',λ') are the geocentric colatitude and geocentric longitude of Q, and the expression of K_n is as follows:

$$K_n=\left(\frac{fM}{R}\right)^2\sum_{m=0}^{n}(\bar{C}_{nm}^{*2}+\bar{S}_{nm}^2) \quad (3)$$

where fM is the gravitational constant, $(\bar{C}_{nm}^{*},\bar{S}_{nm})$ are fully normalized potential coefficients.

According to the addition theorem[1], the series expansion of covariance function of disturbing potential can be expressed as

$$K(P,Q)=\sum_{n=2}^{\infty}\frac{K_n}{2n+1}\frac{R^{n+1}}{r^{n+1}}\frac{R^{n+1}}{(r')^{n+1}}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\bar{P}_{nm}(\cos\theta')\cos m(\lambda-\lambda') \quad (4)$$

3 Traditional formulae of variance and covariance function of disturbing gravity gradient tensors

The relationship between disturbing potential and disturbing gravity gradient tensors is[5-9]

$$T_{zz}=\frac{\partial^2T}{\partial r^2}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}[\bar{C}_{nm}^{*}\cos m\lambda+\bar{S}_{nm}\sin m\lambda]\,[(n+1)(n+2)\bar{P}_{nm}(\cos\theta)] \quad (5)$$

$$T_{xx}=\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2T}{\partial\theta^2}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}(\bar{C}_{nm}^{*}\cos m\lambda+\bar{S}_{nm}\sin m\lambda)\left[\frac{\mathrm{d}^2\bar{P}_{nm}(\cos\theta)}{\mathrm{d}\theta^2}-(n+1)\bar{P}_{nm}(\cos\theta)\right] \quad (6)$$

$$T_{yy}=\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\cos\theta}{r^2\sin\theta}\frac{\partial T}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2T}{\partial\lambda^2}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}[\bar{C}_{nm}^{*}\cos m\lambda+\bar{S}_{nm}\sin m\lambda]\left[\frac{\mathrm{d}\bar{P}_{nm}(\cos\theta)}{\mathrm{d}\theta}\frac{\cos\theta}{\sin\theta}-\frac{m^2\bar{P}_{nm}(\cos\theta)}{\sin^2\theta}-(n+1)\bar{P}_{nm}(\cos\theta)\right] \quad (7)$$

$$T_{xy}=\frac{1}{r^2\sin\theta}\frac{\partial^2T}{\partial\theta\partial\lambda}-\frac{\cos\theta}{r^2\sin^2\theta}\frac{\partial T}{\partial\lambda}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}[\bar{C}_{nm}^{*}\sin m\lambda-\bar{S}_{nm}\cos m\lambda]\left[\frac{m\cos\theta}{\sin^2\theta}\bar{P}_{nm}(\cos\theta)-\frac{m}{\sin\theta}\frac{\mathrm{d}\bar{P}_{nm}(\cos\theta)}{\mathrm{d}\theta}\right] \quad (8)$$

$$T_{xz}=\frac{1}{r^2}\frac{\partial T}{\partial\theta}-\frac{1}{r}\frac{\partial^2T}{\partial r\partial\theta}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}[\bar{C}_{nm}^{*}\cos m\lambda+\bar{S}_{nm}\sin m\lambda]\left[(n+2)\frac{\mathrm{d}\bar{P}_{nm}(\cos\theta)}{\mathrm{d}\theta}\right] \quad (9)$$

$$T_{yz}=\frac{1}{r^2\sin\theta}\frac{\partial T}{\partial\lambda}-\frac{1}{r\sin\theta}\frac{\partial^2T}{\partial r\partial\lambda}=\frac{fM}{r^3}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}[\bar{C}_{nm}^{*}\sin m\lambda-\bar{S}_{nm}\cos m\lambda]\left[-(n+2)\frac{m\bar{P}_{nm}(\cos\theta)}{\sin\theta}\right] \quad (10)$$