Geomorphology 215 (2014) 74–82

Contents lists available at ScienceDirect

Geomorphology

journal homepage: www.elsevier.com/locate/geomorph

Morphodynamic reaction of a schematic river to sediment input changes: Analytical approaches

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ARTICLE INFO

Article history:
Received 31 March 2012
Received in revised form 13 May 2013
Accepted 28 May 2013
Available online 5 June 2013

Keywords:
Sediment transport
Large rivers
Stepwise forcing
Periodical forcing
Response Time
Adaptation Length

ABSTRACT

The paper investigates the morphodynamic reaction of a schematic river to a perturbation of the sediment input imposed at its upstream end. The consequent evolution of the initially equilibrium river can be studied by means of various models (0-dimensional; 1-D parabolic and 1-D hyperbolic with uniform sediments; 1-D hyperbolic with graded sediments), depending on the more or less simplified differential equations applied for describing the water and sediment motion.

The paper discusses a number of analytical solutions obtained with two types of boundary conditions, namely: (i) stepwise change of the sediment input, very often connected to anthropogenic actions, and (ii) sinusoidal input variation with prescribed period, typically associated to the meteorological cycles (short period) or to the geological and climate change (long period). The solutions provide an explicit expression for the Response Time of the river for the condition (i) and the Attenuation Length of the river for the condition (ii). The two quantities as defined in this paper do have a specific physical meaning, strictly connected with the corresponding boundary conditions, which measures the reactivity of the river to external disturbances.

A comparative application of the analytical solutions to various (prevalently large) rivers of the world is given, together with a final discussion.

1. Introduction

Following appreciable variations of boundary conditions, due to either natural (e.g. climate change) or anthropogenic (e.g. water extraction, damming or sediment mining) causes, rivers may experience long-term morphological evolution, which will eventually affect large portions of their watersheds.

Depending on the extent of the affected zone and the rapidity of the reactions, counter measures to be applied are necessarily different, substantially falling in three categories. For relatively concentrated changes in time and space, one may try to contrast directly their causes. For more persistent and/or spread out variations, possible mitigation or compensation of their negative effects may be considered. For perturbations which are expected to propagate over extremely long distances along the river for a very long time, the only feasible strategy is preparing the system for a progressive adaptation to its final conditions.

It may then be reasonable to introduce an overall characteristic parameter of each river system, to be added to the many others (topographical, morphometric and hydrological) already used for their "synchronic" classification. This parameter should be particularly significant in "diachronic" terms, namely of morphodynamic evolution. An interesting parameter in this respect is the so-called Response Time, which could be defined as the time required by the river to reduce to a prescribed fraction (e.g. 1/2 or 1/e) the punctual perturbation applied at the upstream end over the entire river length.

For some large rivers, the Response Time for the entire watercourse defined in this way is in the range of 10^3–10^5 years (Dade and Friend, 1998). A systematic assessment of the Response Time for many tens of the largest rivers in the world was made, following Castelltort and Van Den Driessche (2003), by Gupta (2007), based on a diffusive parabolic equation as suggested by various authors (Allen and Densmore, 2000).

It is important to recall that the terms parabolic, hyperbolic and elliptic (not mentioned here) follow from the classification of the partial differential equations that describe a wide variety of phenomena such as fluid flow, sediment transport, heat transport, elasticity, etc. In particular, the partial differential equations that describe in one-dimensional terms the water and sediment movement along a river have a hyperbolic character. This implies that water and sediment disturbances created in the river system propagate, in the upstream and downstream directions, with a finite celerity (i.e. in a definite time). Moreover, if the transported material is assumed to be poorly sorted, each sediment grain size class gives rise to a different propagating disturbance. The corresponding partial differential equations will constitute the hyperbolic (complete) one-dimensional (1-D) model with sorted material.

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http://dx.doi.org/10.1016/j.geomorph.2013.05.021

In the engineering practice, the hyperbolic 1-D model (both with uniform and sorted materials) has been numerically applied to a great number of real rivers, sometimes accounting for their transversal dynamics (Nones et al., in press). The complete model with sorted material has also been experimentally verified by laboratory experiments (Cui et al., 2003a,b; Wright and Parker, 2005).

It should be noted, however, that the complete model (especially with sorted material) is excessively cumbersome for long-time, large-scale numerical simulations. It is therefore interesting to assess the possibility of opportune simplifications. The most common of these simplifications is assuming that water slopes, energy slope and bottom slope coincide when averaged over a sufficiently long river reach. Under this hypothesis (LUF or Local Uniform Flow), the complete hyperbolic equations with uniform grain size material become of parabolic type and are much easier to handle both from the numerical and analytical points of view. In the parabolic model (like in heat diffusion) the disturbances present a spatial damping, but propagate along the river with an infinite celerity (i.e. instantaneous propagation). Thus its solution may strongly differ, in some cases, from the solution provided by the complete (hyperbolic) model. Since the pioneering work of de Vries (1975), the discussion of the hyperbolic model and its possible simplifications has been an important issue of the river engineering research.

The 1-D mathematical model, either in its complete form or conveniently simplified, is applied in this paper to evaluate the Response Time as well as the other quantities defined later for assessing the morphodynamic reaction of a schematic river to sediment input changes.

As in many of the papers cited above, the schematic river is represented by an initial (equilibrium) linear bottom profile having the length L and a constant slope $I_0 = H_0/L$, with H_0 the initial relief of the watershed. The inputs of water and sediment, either with a uniform grain size or with an extended granulometry, are assumed concentrated at the upstream end of the river. Consequently, in absence of a progressive feeding along the river from the tributaries (and, ultimately, from the watershed slopes), the river width is assumed as a constant, called B.

This is undoubtedly a very crude schematization, but all the same necessary to achieve possible analytical solutions, providing a direct evaluation of the long-term morphodynamics over the entire rivers while permitting to compare among them the different rivers in relative terms. By such a schematization, moreover, the 1-D partial differential equations mentioned before can be further transformed into a 0-D ordinary differential equation. The 0-D model has only the time as independent variable and provides an even simpler solution for the morphodynamic reaction of the river.

The quantification of the Response Time as defined above refers to a stepwise perturbation; namely the sudden change from a certain (constant) value to another (also constant) value of the sediment fluxes entering the river system from the watershed slopes, but assumed concentrated at the upstream boundary of the river. This perturbation is very often connected to an anthropogenic action: it may simulate, for example, the effects of a sudden reduction or increase of sediment production, respectively caused by the damming of an important tributary or by a rapid deforestation of the basin surface.

By contrast, a different type of perturbation having a cyclic character is typically associated to natural causes: for example, the change of sediment input due to seasonal variations (with the oscillation period of the relevant flood waves, up to one year) or due to much longer periodicities (up to millennia or more), like the geological ones. Periodical forcing involves a different reaction quantity of the river, called Attenuation Length.

In the next sections it shall provide a brief description of various models, more or less sophisticated, together with some of their possible analytical solutions for either significant boundary condition.

The models considered in this paper are classified below, according to the following features:

- spatial dimensions: zero-dimensional (0-D) and one-dimensional (1-D);
- type of perturbation at the upstream boundary: stepwise and sinusoidal;
- fundamental equations: parabolic (LUF) and hyperbolic (complete);
- grain size composition of the transported sediments: uniform and sorted.

A verification of the various simplified models has been made via their mutual comparison and, ultimately, against the (complete) hyperbolic model with sorted materials.

2. Zero-dimensional model

In Figure 1 we report the generic schematization of the 0-D model of a river, corresponding to the configuration mentioned in the Introduction section. The 0-D approach assumes that the morphological characteristics of the river, although varying in space, are fully represented by the time-dependent bottom slope at the downstream end, assumed to be the same along the entire watercourse. The analysis assumes that the river evolution starts from an initial configuration (indicated with the subscript 0) and changes during the time t. As in other models, the hypothesis made is that the river length L and the width B remain constant during the entire evolution.

The subsequent necessary linearizations, the maximum elevation $H(t)$ of the river bottom, and the variable during the time t, are expressed in terms of small deviation $H'(t)$ from the initial equilibrium condition H_0.

$$H(t) = H_0 + H'(t) = H_0 \cdot \left[1 + \frac{H'(t)}{H_0}\right]. \tag{1}$$

In the same manner, the bottom slope $I(t)$ of the river, is expressed by the equilibrium slope I_0 and a time-variable deviation $I'(t)$.

$$I(t) = I_0 + I'(t) = I_0 \cdot \left[1 + \frac{I'(t)}{I_0}\right]. \tag{2}$$

For computing the annual sediment transport $Q_s(t)$ along the river, we use a monomial equation of the Engelund–Hansen type.

$$Q_s(t) = \alpha' \cdot \frac{Q^m I(t)^n}{B^p d^q} \tag{3}$$

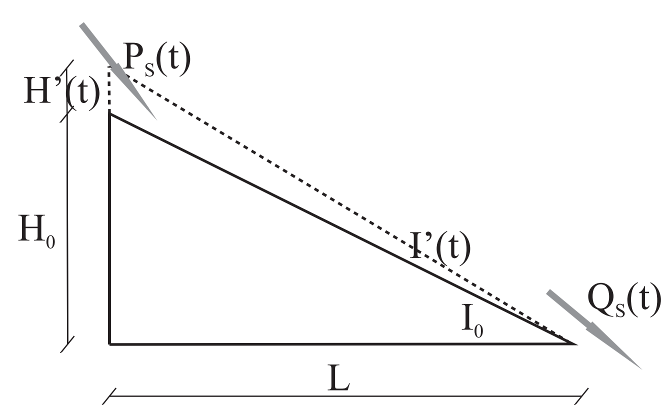

Fig. 1. Scheme of the 0-D model.