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# Computing American option price under regime switching with rationality parameter

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ABSTRACT

American put option pricing under regime switching is modelled by a system of coupled partial differential equations. The proposed model combines better the reality of the market by incorporating the regime switching jointly with the emotional behaviour of traders using the rationality parameter approach recently introduced by Tågholt Gad and Lund Petersen to cope with possible irrational exercise policy. The classical rational exercise is recovered as a limit case of the rational parameter. The resulting nonlinear system of PDEs is solved by a weighted finite difference method, also known as $\theta$-method. In order to avoid the need of an iterative method for nonlinear system, the term with rationality parameter and the coupling term are treated explicitly. Next, the resulting linear system is solved by Thomas algorithm. Stability conditions for the numerical scheme are studied by using von Neumann approach. Numerical examples illustrate the efficiency and accuracy of the proposed method.



## 1. Introduction

In financial derivatives pricing problems, when the stochastic process for the underlying asset is too simple, as when assuming constant parameters [1], the model does not replicate the market price. This drawback has been overcome in the literature by introducing stochastic volatility, jump–diffusion and regime switching models for the underlying price evolution.

Since the paper of Buffington and Elliot [2] the switching model has attracted much attention, mainly due to its ability to model non-constant real scenarios when market switches from time to time among different regimes. It is well known that regime switching models are computationally inexpensive when compared to stochastic volatility jump–diffusion models and provide versatile applications in other fields, like electricity markets [3], valuation of stock loans [4], forestry valuation [5], natural gas [6] and insurance [7].

In this paper we consider a continuous time Markov chain $\alpha_t$ taking values among different regimes, where $I$ is the total number of regimes considered in the market. Thus, each regime is labelled by an integer $i$ with $1 \le i \le I$. Hence, the regime space of $\alpha_t$ is $\Omega = \{1, 2, \ldots, I\}$. Let $Q = (q_{i,j})_{I\times I}$ be the given generator matrix of $\alpha_t$. Following [8], the entries $q_{i,j}$ satisfy:

$$q_{i,j} \ge 0, \quad \text{if } i \ne j; \qquad q_{i,i} = -\sum_{j\ne i} q_{i,j}, \quad 1 \le i \le I. \tag{1}$$

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Under the risk-neutral measure, see Elliot et al. [9] for details, the stochastic process for the underlying asset $S_t$ satisfies the following stochastic differential equation:

$$\frac{dS_t}{S_t} = r_{\alpha_t} dt + \sigma_{\alpha_t} d\tilde{B}_t, \quad t \leq 0, \tag{2}$$

where $\sigma_{\alpha_t}$ is the volatility of the asset $S_t$, $r_{\alpha_t}$ is the risk-free interest rate, both depending on the Markov chain $\alpha_t$, and $\tilde{B}_t$ is a standard Brownian motion defined on some given risk-neutral probability space, independent on the Markov chain $\alpha_t$.

Here we consider the American put option on the asset $S_t = S$ with strike price $E$ and maturity $T < \infty$. For $1 \leq i \leq I$, let $V_i(S, \tau)$ denote the option price functions (i.e.

$$(V_i)_\tau = V_i(S_\tau, \tau)$$

is the option price process in regime $i$), where $\tau = T - t$ denotes the time to maturity and the regime $\alpha_t = i$. Then, $V_i(S.\tau)$, $1 \leq i \leq I$, satisfy the free boundary value problem for $0 < \tau \leq T$, see [10]:

$$\begin{cases} \dfrac{\partial V_i}{\partial \tau} = \dfrac{\sigma_i^2}{2} S^2 \dfrac{\partial^2 V_i}{\partial S^2} + r_i S \dfrac{\partial V_i}{\partial S} - r_i V_i + \displaystyle\sum_{l \neq i} q_{i,l}(V_l - V_i), \quad S > S_i^*(\tau), \\ V_i(S, \tau) = E - S, \quad 0 \leq S \leq S_i^*(\tau), \end{cases} \tag{3}$$

where $S_i^*(\tau)$ denotes the optimal stopping boundary of the option under regime $i$. Initial conditions are

$$V_i(S, 0) = \max(E - S, 0), \qquad S_i^*(0) = E, \quad i = 1, \ldots, I. \tag{4}$$

Boundary conditions for $i = 1, .., I$ are as follows

$$\lim_{S \to \infty} V_i(S, \tau) = 0, \tag{5}$$

$$V_i(S_i^*(\tau), \tau) = E - S_i^*(\tau), \tag{6}$$

$$\frac{\partial V_i}{\partial S}(S_i^*(\tau), \tau) = -1. \tag{7}$$

Several different numerical methods for solving problem (3) have been proposed. Lattice methods [11,12] are popular for practitioners because they are easy to implement. The penalty method [13,10,14] adds a penalty term into each equation of the coupled system. After considering American options pricing under regime switching model as a Hamilton Jacobi Bellman problem, in [15] iterated optimal stopping [16] and local policy iteration [17] methods are compared.

Recently, in [18] the front-fixing method (see [19]) has been employed for valuation of American option under regime switching model, by incorporating free boundary into the PDE as a new unknown variable. In such paper efficient explicit finite difference methods are shown.

Unlike the direct approach of a European option pricing problem where the price is given by the solution of a partial differential equation (PDE), it is well known that the price of an American option is described by the solution of partial differential inequality (see [20]). Once the inequality has been discretized, a linear complementarity problem (LCP) arises with the additional algebraic complexity. Although the LCPs are satisfactory addressed (see [21] and the references therein), the possibility of computing an American option pricing problem by solving a PDE problems could be interesting not only from the computational point of view, but also from the reliability of the computed price.

On the other hand, very recently the rationality parameter approach proposed by Tågholt Gad and Lund Pedersen in [22] allows to incorporate the possibility of an irrational exercise policy in the American option. In this setting, the computation of the price can be obtained by solving a PDE problem with an additional nonlinear term in the corresponding European option pricing formulation. Moreover, when the rationality parameter in the previous model tends to infinity, the formulation tends to the one of the classical American option pricing problem with rational exercise. Therefore, by addressing the solution of the model for large enough values of the rationality parameter provides an approximation of the classical American option price. In the absence of regime switching, the numerical solution of American option pricing models with irrational exercise has been recently addressed [23].

The main aim of this paper is to propose a new model that simultaneously incorporates the advantages of the regime switching jointly with those of the rationality parameter approach. Additionally, we propose and develop the numerical analysis of a suitable family of weighted finite differences schemes to solve numerically this nonlinear model.

The plan of the work is as follows. In Section 2, the new model that takes into account the irrational behaviour under regime switching is described and a suitable change of variables and unknown transforms the original PDE problem into an equivalent one with constant coefficients in the differential part. Section 3 deals with the construction of a one parameter family of finite difference methods, also known as weighted schemes [24]. Next, relevant numerical analysis issues as positivity, stability and consistency are studied in Section 4. Numerical simulations are included in Section 5, paying special attention to the limit case of classical American options with rational exercise, also the order of convergence and the comparison with other methods is presented. Section 6 contains the concluding remarks.