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A comparative study of lattice Boltzmann models for incompressible flow

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A B S T R A C T

For incompressible flow, a comparative study on the four lattice Boltzmann (LB) models, the standard model, the He–Luo model, Guo's model, and the present model, is performed. Theoretically, the macroscopic equations derived from the involved LB models are compared by the Chapman–Enskog analysis. Then, the analytical framework proposed in M. Junk's work is applied to investigate the finite difference stencils and the equivalent moment systems pertaining to the concerned LB models. Conclusions are drawn from the theoretical derivations that the truncated error terms, which differ among the concerned LB models, have effects on the accuracy of the modeled deviatoric stress. Moreover, the cavity flow in two dimensions is adopted as a benchmark test to confirm the theoretical demonstrations. The resulting velocity fields from the present model are more in line with the reference solutions in the region of high deviatoric stress than other three LB models, which is consistent with the theoretical expectations and is further confirmed by the comparisons of the truncation error terms. In addition, we also conclude from the numerical tests that the present model has the advantage of better convergence efficiency but suffers from the worse stability.

1. Introduction

For the study of the hydrodynamics, the simplified micro-dynamical models provide convenient alternatives to the conventional computational fluid dynamics (CFD) methods based on the direct discretization of the Navier–Stokes (N–S) equations. The lattice gas automata (LGA) and the lattice Boltzmann equation (LBE), with evolution rules preserving the mass and momentum conservation, belong to such models. In particular, the LBE can be derived from the lattice gas micro-dynamics by replacing the Boolean variables in the LGA with the real-valued distribution function [1–3]. Therefore, the LBE method is expected to be more computationally efficient than the LGA since the statistical noise arising from the Boolean computing in the LGA is eliminated [1–3]. Moreover, with the further simplification of the collision operator [3–6], the resulting LB model is widely adopted in the simulation of the practical flow.

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According to the Chapman–Enskog (CE) analysis [7], the macroscopic N–S equations can be recovered from the LBE under the low Mach number (Ma) limit [8–13], making the lattice Boltzmann method applicable widely to incompressible flow. Specifically, the form of the recovered macroscopic equations largely depends on the expression of the equilibrium distribution function (EDF). With the EDF of the standard LB model [14], isothermal, compressible N–S equations, which approximate the incompressible flow in second-order truncation error of Ma, are obtained [15]. To recover directly into the incompressible N–S equations, many efforts, in constructing an incompressible LB model, have been made. By reconstructing the EDF, a LB model for incompressible steady flow was realized in 1995 [11,12]. Thereafter, another incompressible LB model, decomposing the fluid density ρ into the mean density ρ_0 and the density fluctuation $\delta\rho$, is proposed by He and Luo [13], and denoted as He–Luo model in this paper. Ignoring the effect of $\delta\rho$ in $\rho\mathbf{u}$, the exact N–S equations for steady incompressible flow are derived from the He–Luo model. However, similar to the standard LB model, the fluid pressure in the He–Luo model is related to the density through the ideal gas state equation. And such dependence of density on the pressure makes the recovered continuum equation different from the standard continuum equation for unsteady incompressible flow.

Alternatively, the EDF of the incompressible LB model, proposed by Guo [10], is defined as a function of the fluid pressure and velocity. By assuming the fluid density to be a constant and independent of the pressure, the N–S equations for incompressible flow are recovered from Guo's model. While in the present LB model, the moments of the EDF are firstly revised (referring to Eqs. (40)–(42)), then, the corresponding EDF is derived from the moment expansion proposed by Grad [16–19]. Similar to Guo's model, the fluid density in the present model is prescribed to be a constant, and thus the incompressible N–S equations are also derived from the present model. The main differences between the current two models, the present model and Guo's model, are the expression of the fluid pressure, which is derived from the diagonal part of the momentum flux tensor in the present model, while which is derived from the expression of the $f_0^{(0)}$ in Guo's model. Moreover, in Guo's model, an approximation, which may reduce the accuracy of the deviatoric stress, is introduced in deriving pressure formula. Therefore, a more accurate evaluation of the deviatoric stress is theoretically expected for the present model.

Differing from the CE analysis in the Boltzmann scale (i.e. the convective scale), an analytical framework in the N–S scale (i.e. the diffusive scale) proposed in M. Junk's work [20,21] is adopted to further compare the concerned LB models. The finite difference interpretations pertaining to these models are derived and the adopted finite difference stencils are compared. Then, the LBE in the diffusive scaling is expanded, which leads to a modified Boltzmann equation. The equivalent moment systems, pertaining to such modified Boltzmann equation, demonstrate that the Laplacian of the zero order moment of the distribution function ΔM_0 has an explicit effect on the accuracy of the modeled stress tensor. Thus, the convergence and distribution of ΔM_0 are investigated in Section 3 of this work.

When applying the standard LB model, Guo's model and the He–Luo model to practical incompressible flow, take the cavity flow for example, the general flow features such as the location and strength of the vortexes are well acquired [10,22,23]. However, visible deviations of the resulting velocity fields are reported in the previous work [23–30,41], especially when a relatively low grid resolution is applied. Attempts have been made to improve the accuracy of the LB simulations in these regions, including the multi-block technique [24,25], stencil adaptive technique [27] and the nonuniform quadtree grids [28]. In this work, the performances of the concerned LB models are compared with access to such representative benchmark test.

2. The incompressible LB models

2.1. The lattice Boltzmann equation

The lattice Boltzmann BGK equation is

$$f_\alpha(\mathbf{x}+\boldsymbol{\xi}_\alpha dt, t+dt) - f_\alpha(\mathbf{x}, t) = -\frac{1}{\tau}[f_\alpha(\mathbf{x}, t) - f_\alpha^{(0)}(\mathbf{x}, t)] \tag{1}$$

where τ is the relaxation time; $f_\alpha(\mathbf{x}, t)$ and $f_\alpha^{(0)}(\mathbf{x}, t)$ are the discrete distribution function and its equilibrium version, respectively. The discrete velocity vectors $\boldsymbol{\xi}_\alpha$ in Eq. (1), for 2D incompressible flow, are defined as below:

$$\boldsymbol{\xi}_\alpha = \begin{cases} (0,0) & \alpha = 0 \\ c(\cos[(\alpha-1)\pi/2], \sin[(\alpha-1)\pi/2]) & \alpha = 1,2,3,4 \\ \sqrt{2}c(\cos[(2\alpha-1)\pi/4], \sin[(2\alpha-1)\pi/4]) & \alpha = 5,6,7,8 \end{cases} \tag{2}$$

where $c = dx/dt$, and dx, dt are the spatial and time step, respectively. Additionally, in the present numerical simulations, dx and dt are both set as the dimensionless constant 1, i.e. $c = 1$.

2.2. The standard LB model

In the standard LB model [14], the discrete EDF $f_\alpha^{(0)}$ is determined as

$$f_\alpha^{(0)} = \rho w_\alpha \left\{ 1 + \frac{\boldsymbol{\xi}_\alpha \bullet \mathbf{u}}{RT} + \frac{(\boldsymbol{\xi}_\alpha \bullet \mathbf{u})^2}{2(RT)^2} - \frac{\mathbf{u}^2}{2RT} \right\} \tag{3}$$