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A wavelet multiscale-homotopy method for the parameter identification problem of partial differential equations

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ABSTRACT

By introducing the wavelet multiscale method and the homotopy method to the inversion process for the parameter identification problem of partial differential equations, a joint inversion method called the wavelet multiscale-homotopy method is proposed, which is globally convergent, computationally efficient, and has the anti-noise ability. As a practical problem, the parameter identification problem of the saturation equation in the fractional flow formulation of the two-phase porous media flow equations is solved. Numerical results validate the method's merits.

1. Introduction

Consider the following parameter identification problem of partial differential equations

$$\begin{cases} L(p(x),t)u(x,t) = s(x,t), & x \in \Omega,\ 0 \le t \le T, \\ Eu(x,0) = h(x), & x \in \Omega, \\ Bu(x,t) = g(x,t), & x \in \partial\Omega,\ 0 \le t \le T, \\ Au(x,t) = f(x,t), & x \in \Gamma \subset \Omega,\ 0 \le t \le T, \end{cases} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^\top$, $\Omega \subset \mathbb{R}^n$ is a bounded domain, $\partial\Omega$ is the boundary of Ω, Γ is a part of Ω, $u(x,t)$ is a sufficiently smooth function, and $s(x,t)$ is a piecewise smooth source function. L, E, B and A are differential operator, initial condition operator, boundary condition operator and auxiliary condition operator, respectively. $p(x)$ is the parameter to be identified in L.

As $u(x,t)$ nonlinearly depends on $p(x)$, a nonlinear operator $J(p)$ can be defined as

$$J(p) = Au(p; x, t) - f(x,t) \quad x \in \Gamma \subset \Omega,\ 0 \le t \le T, \tag{2}$$

then the identification for the parameter $p(x)$ is reduced to the output least squares problem

$$\min \|J(p)\|^2. \tag{3}$$

Generally speaking, the problem of (3) presents a number of difficult challenges, due to its ill-posedness, nonlinearity and large computational cost. First, the solution does not depend continuously on the measurement data, that is, a minor disturbance of the measurement data may cause large change on the solution. Second, nonlinear dependence of the measurement data with respect to the solution causes the presence of numerous local minima, so a good initial estimate is

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crucial for any numerical methods. Finally, the forward model is described by the solution of a partial differential equation which is computationally demanding to solve. So the key issue is how to quickly find a stable solution in a wide range.

In order to overcome these difficulties, this paper combines the wavelet multiscale method with the homotopy method, so as to propose a wavelet multiscale-homotopy method for the general parameter identification problem of partial differential equations.

Multiscale method was first studied to solve the forward problem, for example, the 3D convection–diffusion equation [1], the convection–diffusion–reaction problems [2], the two-phase immiscible flow simulations in heterogeneous porous media [3,4], and then was extended to the inverse problem [5]. As a special class of multiscale methods, wavelet multiscale method has recently emerged in the field of inversion. Fu et al. [6] presented a wavelet multiscale method for identifying the velocity in a two-dimensional acoustic wave equation. Ding et al. [7] presented a wavelet multiscale method for identifying the conductivity in Maxwell equations. Zhang et al. [8] presented a wavelet multiscale method for identifying the porosity in a fluid-saturated porous media. Chiao et al. [9] presented a wavelet multiscale procedure for identifying the parameter of non-linear geophysics. For electrical capacitance tomography [10] and diffuse optical tomography [11], wavelet multiscale method was shown to be very effective. These works showed that the wavelet multiscale method can enhance stability of inversion, accelerate convergence and cope with the presence of local minima to reach the global minimum. The rationale behind this success is that the cost function shows stronger convexity and has less minima at longer scale such that the global minimum can be achieved. Therefore, best results can be obtained, using this solution to the initialization of the optimization problem, at the shorter scale until finding the minimum at the original scale.

Homotopy method provides an ideal tool for solving the nonlinear problems due to its widely convergent property [12–14]. It is a very interesting research opportunity to use homotopy method to solve inverse problems. Successful applications of this method include the two-phase inverse Stefan problem [15], the inversion of the elliptical equation [16], the PEM identification of ARMAX models [17], the inverse heat conduction problem [18], the parameter estimation of the nonlinear diffusion equation [19], the inversion of the two-dimensional acoustic wave equation [20] and the well-log constraint waveform inversion [21]. All these works showed the effectiveness of homotopy method on the inverse problems.

The rest of this paper is organized as follows. Section 2 derives the homotopy method for the general parameter identification problem. Section 3 presents the wavelet multiscale-homotopy method. Section 4 provides the application to the parameter identification problem of the saturation equation in the fractional flow formulation of the two-phase porous media flow equations. The conclusions of this paper are summarized in Section 5.

2. Homotopy method

Since the problem (3) is ill-posed, regularization method must be employed. In Tikhonov regularization, the problem (3) is replaced by the minimization problem

$$\|J(p)\|^2 + \alpha\|p - p^0\|^2 \to \min, \tag{4}$$

where α is the regularization parameter, p^0 is an initial estimate for a solution p^* of (2). The iteratively regularized Gauss–Newton method (see [22])

$$p^{k+1} = p^k - [J'(p^k)^*J'(p^k) + \alpha I]^{-1}[J'(p^k)^*J(p^k) + \alpha(p^k - p^0)], \quad k = 0, 1, \ldots \tag{5}$$

can be used to solve (4). It has fast convergence speed and good stability, however, it is a locally convergent scheme. To overcome this weakness, we consider to introduce the homotopy method.

It is easy to know that (4) is equivalent to the corresponding normal equation

$$J'(p)^*J(p) + \alpha(p - p^0) = 0. \tag{6}$$

To solve this normal equation, the fixed point homotopy equation is considered

$$H(p, \mu) = \mu[J'(p)^*J(p) + \alpha(p - p^0)] + (1 - \mu)(p - p^0) = 0, \tag{7}$$

where $\mu \in [0, 1]$ is the homotopy parameter, p^0 is an arbitrary initial estimate. Divide the interval $[0, 1]$ into $0 = \mu_0 < \mu_1 < \cdots < \mu_N = 1$, and for $\mu = \mu_k$, use some iterative method to solve (7) sequentially. As the solution p^0 of $H(p, \mu_0) = 0$ is known, it can be taken as the initial approximation of the next equation $H(p, \mu_1) = 0$. Assume that the approximation solution p^k of $H(p, \mu_k) = 0$ has already been found. By linearization method, $H(p, \mu_{k+1}) = 0$ can be solved with $J(p)$ replaced by $J'(p_j^k)(p - p_j^k) + J(p_j^k)$. Then, the iterative formula is given as follows

$$\begin{aligned} p_{j+1}^k &= p_j^k - [\mu_k J'(p_j^k)^*J'(p_j^k) + (1 - \mu_k + \alpha\mu_k)I]^{-1}[\mu_k J'(p_j^k)^*J(p_j^k) + (1 - \mu_k + \alpha\mu_k)(p_j^k - p^0)], \\ &\quad j = 0, 1, \ldots, k_m,\ p_0^k = p^{k-1},\ p^k = p_{k_m+1}^k,\ k = 1, 2, \ldots, N. \end{aligned} \tag{8}$$

If $\mu_k = \frac{k}{N}$ by an isometric division and $k_m \equiv 0$, then (8) becomes a more simple formula

$$\begin{aligned} p^{k+1} &= p^k - \left[\frac{k}{N}J'(p^k)^*J'(p^k) + \left(1 - \frac{k}{N} + \alpha\frac{k}{N}\right)I\right]^{-1}\left[\frac{k}{N}J'(p^k)^*J(p^k) + \left(1 - \frac{k}{N} + \alpha\frac{k}{N}\right)(p^k - p^0)\right], \\ &\quad k = 0, 1, \ldots, N - 1. \end{aligned} \tag{9}$$