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A new efficient numerical method for solving American option under regime switching model

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ABSTRACT

A system of coupled free boundary problems describing American put option pricing under regime switching is considered. In order to build numerical solution firstly a front-fixing transformation is applied. Transformed problem is posed on multidimensional fixed domain and is solved by explicit finite difference method. The numerical scheme is conditionally stable and is consistent with the first order in time and second order in space. The proposed approach allows the computation not only of the option price but also of the optimal stopping boundary. Numerical examples demonstrate efficiency and accuracy of the proposed method. The results are compared with other known approaches to show its competitiveness.

1. Introduction

Valuation of derivatives used to be based on the assumption of a stochastic process for the underlying asset and the construction of a dynamic, self-financing hedging portfolio to minimize the uncertainty (risk). Using the absence of arbitrage principle, the initial cost of constructing the portfolio, typically given by a partial differential equation (PDE), is then considered to be the fair value of the derivative, [1].

When the stochastic process for the asset is too simple, assuming constant parameters, like [2] the model does not replicate the market price. This drawback has been overcome with stochastic volatility, jump diffusion and regime switching models.

Since Buffington and Elliot's seminal paper [3] the switching model has attracted much attention due to its capacity of modelling non-constant real scenarios when market switches from time to time among different regimes.

Furthermore, regime switching models are computationally inexpensive compared to stochastic volatility jump diffusion models and have versatile applications in other fields, like electric markets [4], valuation of stock loans [5], forestry valuation [6], natural gas [7] and insurance [8].

In this paper we consider a continuous time Markov chain α_t taking values among I different regimes, where I is the total number of regimes considered in the market. Each regime is labelled by an integer i with $1 \le i \le I$. Hence, the regime space of α_t is $M = \{1, 2, \dots, I\}$. Let $Q = (q_{i,j})_{I\times I}$ be the given generator matrix of α_t. From [9] the entries $q_{i,j}$ satisfy:

$$q_{i,j} \le 0, \quad \text{if } i \ne j; \qquad q_{i,i} = -\sum_{j\ne i} q_{i,j}, \quad 1 \le i \le I. \tag{1}$$

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Under the risk-neutral measure, see Elliot et al. [10] for details, the stochastic process for the underlying asset S_t is

$$\frac{dS_t}{S_t} = r_{\alpha_t} dt + \sigma_{\alpha_t} d\tilde{B}_t, \quad t \le 0, \tag{2}$$

where σ_{α_t} is the volatility of the asset S_t and r_{α_t} is the risk-free interest rate.

Here we consider the American put option on the asset $S_t = S$ with strike price E and maturity $T < \infty$. Let $V_i(S, \tau)$ denote the option price functions, where $\tau = T - t$ denotes the time to maturity, the asset price S and the regime $\alpha_t = i$. Then, $V_i(S.\tau)$, $1 \le i \le I$, satisfy the following free boundary problem:

$$\frac{\partial V_i}{\partial \tau} = \frac{\sigma_i^2}{2} S^2 \frac{\partial^2 V_i}{\partial S^2} + r_i S \frac{\partial V_i}{\partial S} - r_i V_i + \sum_{l \ne i} q_{il}(V_l - V_i), \quad S > S_i^*(\tau),\ 0 < \tau \le T, \tag{3}$$

where $S_i^*(\tau)$ denote optimal stopping boundaries of the option. Initial conditions are

$$V_i(S, 0) = \max(E - S, 0), \qquad S_i^*(0) = E, \quad i = 1, \dots, I. \tag{4}$$

Boundary conditions for $i = 1, .., I$ are as follows:

$$\lim_{S \to \infty} V_i(S, \tau) = 0, \tag{5}$$

$$V_i(S_i^*(\tau), \tau) = E - S_i^*(\tau), \tag{6}$$

$$\frac{\partial V_i}{\partial S}(S_i^*(\tau), \tau) = -1. \tag{7}$$

Several different numerical methods for solving problem (3) have been proposed. Lattice methods [11,12] are popular for practitioners because they are easy to implement, but they have the drawback of the absence of numerical analysis and subsequent unreliability, because the lack of numerical analysis may waste the best model. The penalty method [1,13–15] uses a coupling of the penalty term and the regime coupling terms. Both, the lattice and penalty methods do not calculate the optimal stopping boundary that has interest from the practitioners point of view.

The challenging task of the free boundary as another unknown into the PDE problem is not new in the literature. In fact, since Landau's ideas [16] the so-called front-fixing method has been used in many fields [17] and by [18–22] for American option problems without switching.

In this paper we address the numerical solution of the coupled PDE system (3). Firstly, in Section 2 by extending the ideas developed in [19], the PDE system (3) is transformed into a new PDE system on a fixed domain where the free boundaries $S_i^*(\tau)$, $1 \le i \le I$, are incorporated as new unknowns of the system. This allows the computation not only of the prices, but also of all the optimal exercise prices.

In spite of the apparent complexity of the transformed problem due to the appearance of new spatial variables, one for each equation, the explicit numerical scheme constructed in Section 3 becomes easy to implement, computationally cheap and accurate when one compares with the more relevant existing methods. Implicit weighted schemes have been developed in this section for the sake of performance comparison.

Stability and consistency of the numerical method are treated in Section 4. Numerical results are illustrated in Section 5. Paper concludes in Section 6.

2. Multivariable fixed domain transformation

Fixed domain transformation techniques inspired in Landau ideas [16] have been used by several authors [23,22,24,19] for partial differential equations modelling American option pricing problems. To our knowledge this transformation technique has not been applied before for a partial differential system with several unknown free boundaries, one for each equation.

Based on the transformation used by the authors in [23,19] for the case of just one equation, let us consider the multivariable transformation

$$x^i = \ln \frac{S}{S_i^*(\tau)}, \quad 1 \le i \le I. \tag{8}$$

Note that the new variables x^i lie in the fixed positive real line. Price V_i of ith regime involved in ith equation of the system and ith free boundary are related by the dimensionless transformation

$$P_i(x^i, \tau) = \frac{V_i(S, \tau)}{E}, \qquad X_i(\tau) = \frac{S_i^*(\tau)}{E}, \quad 1 \le i \le I. \tag{9}$$

Value of option lth regime appearing in ith coupled equation, $l \ne i$, becomes

$$P_{l,i}(x^i, \tau) = \frac{V_l(S, \tau)}{E}. \tag{10}$$