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Topology optimization of incompressible Navier–Stokes problem by level set based adaptive mesh method☆

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ABSTRACT

This paper presents a level set based adaptive mesh method for solving the topology optimization of incompressible Navier–Stokes problem. The objective is to minimize the dissipated power in the fluid, subject to the Navier–Stokes problem as state equations with a fluid volume constraint. The material distribution information that implicitly represented via level set function is considered as the design variable, which provides an easy way to construct the refinement indicator. Shape and topology sensitivity analysis suggest the steepest descent direction. By the proposed method, the computational expense is mainly focused near the interfaces, which lead to a significant reduction of the computational cost. Although illustrated by the Navier–Stokes problem, we would like to emphasize that our method is not restricted to this particular situation, it can be applied to a wide range of shape or topology optimization problems arising from the fluid dynamics.

1. Introduction

Topology optimization has become an effective design tool for obtaining efficient structures and has attracted a large amount of attention from both engineers and mathematicians. Numerous studies have demonstrated that greater savings could be achieved if selected an appropriate layout at a very early stage of the industrial design. However, despite that great advances have been made in the field of optimal shape design methods for fluid dynamic, and many approaches have been introduced and applied to a broad range of design problems [1,2], it is only about ten years since the pioneering works of Borrvall and Petersson [3] that topological optimization has been developed and used in fluid design problems.

Over the last decades, substantial efforts have been made in the development of the topology optimization methods. Homogenization approach, proposed by Bendsoe and Kikuchi [4], is one of the most famous method in dealing with topology optimization problems. The main idea of the homogenization method is to introduce a material model with micro-scale voids and the topology optimization problem is then converted to seek the optimal layout of such a porous medium. Inspired by the same idea presented in [4], another significant algorithm, the so-called SIMP (Solid Isotropic Material with Penalization) method, was established in [5]. In SIMP method, the variables are the element relative densities, and the material properties, which modelled as the relative material density raised to some power times the material properties of solid material, assumed to be constant within each element. Due to its flexibility, generality, and easy to integrate with the existing finite element method software, SIMP method has been well established in structural optimization of solid

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structures and become the most popular methods in recent years. In the field of fluid dynamics, one of the first approaches is proposed by Borrvall and Petersson in [3], in which the relaxed material distribution approach was implemented to minimize the power dissipated in Stokes flow.

Devised and introduced by Osher and Sethian [6], level set method, originally proposed to numerically simulate the motion of interfaces and free boundaries in fluid dynamics, is a simple and versatile substitution for material distribution based method. Moreover, level set method need not parameterize the structural shape and topology and can present the interfaces between adjacent phases continuously, therefore ensuring a smooth boundary. The greatest strength of the level set method is that it can accurately track the interface and change the topology in a very natural way. That is, material is not the necessary factor to treat topological changes any more and the interfaces or boundary was not tracked directly. Level set method can handle topological changes easily during the evolution process, which distinguishes it from the conventional boundary variation methods. This feature has contributed it to the development of new procedures for solving a wide range of scientific and engineering problems, such as fluid mechanics, material sciences, combustion, computer animation and image processing, geometric optics, and biology, etc. [7]. Above all, the level set method has been applied to solve the shape or topology optimization problems and gained great success [8,9].

In many situations, although what we are interested in is only the zero level sets of the level set functions (i.e., the interface or boundary of the domain), the level set function was still defined over the whole computational domain and implemented by the numerical method on the overall uniform grids, which significantly increase the computational cost. To overcome this shortcoming, several methods have been suggested, such as the narrow band method and the local level set method [10,11]. There are also some adaptive methods aimed to improve the efficiency and accuracy in the literature [12–14].

In the present study, we concern ourselves with topology optimization problems in fluid dynamics. The state problem is represented by the Navier–Stokes equations with nontrivial boundary conditions. In order to solve topology optimization of incompressible Navier–Stokes problem, most of all, to achieve higher resolution of the interface with a minimum of additional expense, we present a numerical method using the level set approach, coupled with an adaptive mesh method. In the proposed approach, the inverse of the local permeability of the medium is implicitly coupled with the level set function. The main motivation is the material distribution method. In addition to use the shape derivative of the objective function to evolve the level set function, the topology derivative is incorporated into the level set function, in order to nucleate new holes during the optimization process. During the mesh refinement procedure, a uniform coarse mesh was first constructed over the whole computational domain for evolving the level set function. The level set equation was solved on the uniform meshes, for which standard high-order numerical methods can be employed [15–17]. The values of the level set function on the coarse grid provide rough estimates, and the interface is also represented as the zero level set of a smooth function. As we know, the level set function has steep gradients at the interface. Therefore, we employ the level set function as the refinement indicator, and any coarse mesh cell that indicated by our strategy is further divided into a uniform fine mesh. Refinement process is performed automatically during the optimization procedure. The optimization problem was then solved using the refined mesh. We can see that the computation is mainly localized near the moving boundaries, which can increase the numerical efficiency of the optimization process. At the same time, the adaptive mesh refinement enables us to achieve higher resolution of the interface.

The method we proposed in this paper has at least the following advantages:

- Higher resolution of the interface can be achieved;
- Topological changes can be handled easily, i.e., new holes can be nucleated automatically during the optimization process;
- The computational cost is significantly reduced compared with the uniform mesh method.

The paper consists of 6 sections. After giving the background and explaining the motivation in Section 1, we formulate the general topology optimization problem for fluid flow and give the most concerned objective functional—the power dissipation to be minimized in Section 2; In Section 3 a brief introduction to the level set method, combined with the shape and topology sensitivity analysis, is presented; The main part of the paper, Section 4, is devoted to a level set based adaptive mesh method and numerical realization of the proposed method. In Section 5, the model examples show that very good results can be obtained and that our method can bring a significant contribution to the engineering problems; Finally, conclusions are drawn in Section 6.

2. Topology optimization of the Navier–Stokes problem

The focus of this work is on topology optimization problem that constrained by incompressible Navier–Stokes equations. The objective is to minimize the dissipated power in the fluid, subject to the Navier–Stokes problem as state equations and a fluid volume constraint. The topology optimization problem can be mathematically formulated as a control problem governed by a system of partial differential equations (PDEs). The general topology optimization problem in fluid dynamics can be written in the following form:

$$\text{minimize:} \quad \mathcal{J}(\boldsymbol{u}(\gamma), \gamma) = \nu \int_D \left|\nabla \boldsymbol{u}\right|^2 \mathrm{d}x, \tag{1}$$