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Designing correct fluid hydrodynamics on a rectangular grid using MRT lattice Boltzmann approach

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ABSTRACT

While the lattice Boltzmann method (LBM) has become a powerful numerical approach for solving complex flows, the standard lattice Boltzmann method typically uses a square lattice grid in two spatial dimensions and cubic lattice grid in three dimensions. For inhomogeneous and anisotropic flows, it is desirable to have a LBM model that utilizes a rectangular grid. There were two previous attempts to extend the multiple-relaxation-time (MRT) LBM to a rectangular lattice grid in 2D, however, the resulting hydrodynamic momentum equation was not fully consistent with the Navier–Stokes equation, due to anisotropy of the transport coefficients. In the present work, a new MRT model with an additional degree of freedom is developed in order to match precisely the Navier–Stokes equation when a rectangular lattice grid is used. We first revisit the previous attempts to understand the origin and nature of anisotropic transport coefficients by conducting an inverse design analysis within the Chapman–Enskog procedure. Then an additional adjustable parameter that governs the relative orientation in the energy–normal stress subspace is introduced. It is shown that this adjustable parameter can be used to fully eliminate the anisotropy of transport coefficients, thus the exact Navier–Stokes equation can be derived on a rectangular grid. Our theoretical findings are confirmed by numerical solutions using three two-dimension benchmark problems, *i.e.* the channel flow, the cavity flow, and the decaying Taylor–Green vortex flow. The numerical results demonstrate that the proposed model shows remarkably good performance with appropriate choice of model parameters.

1. Introduction

As an alternative numerical method based on kinetic theory, the lattice Boltzmann method (LBM) has attracted a great deal of attention since its inception about 25 years ago [1,2]. The basic idea is to design a fully discrete version of the Boltzmann equation, with a minimum set of discrete microscopic velocities, that can yield the exact Navier–Stokes equation through the Chapman–Enskog analysis. From a computational viewpoint, the advantages of LBM include its algorithm simplicity, intrinsic data locality (thus straightforward to perform parallel computation), and capability to conveniently incorporate complex fluid–solid and fluid–fluid boundary conditions. Hence, LBM has been widely employed in simulations

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of complex fluid systems, such as multiphase flows [3,4], complex viscous flows with deformable boundary and complex geometry such as porous media [5–7], and micro-scale flows [8,9].

Despite the success of LBM, the standard LBM is restricted to a square grid or hexagonal grid in two spatial dimensions (2D) and a cubic grid in three spatial dimensions (3D). This restriction is aligned with the set of microscopic velocities used and is desirable for model isotropy. However, this could result in a low computational efficiency when the flow field is highly nonuniform, inhomogeneous, and anisotropic, such as a boundary layer flow where the velocity gradients in one spatial dimension is much stronger than the other directions. To alleviate this problem within LBM, different approaches have been developed to allow the use of a nonuniform grid. One approach is to employ an interpolation method to decouple the grid associated with lattice Boltzmann microscopic velocities, from the numerical mesh. Pioneering work in this direction includes the studies of Filippova and Hänel [10,11] who reconstructed distribution functions at arbitrary locations using spatial and temporal interpolations. Another approach is to introduce local grid refinement or use different mesh densities for different regions of the flow (*i.e.*, multi-block methods) [12]. Methods to communicate distribution functions, defined on coarse and fine grids, at the block interfaces have been developed. Other methods to use a non-uniform grid typically utilize a local interpolation scheme [13,14]. While these approaches have been actively extended and applied in many applications, their accuracy is limited by the interpolation scheme which may also introduce additional artificial dissipation. Therefore, it is desirable to construct a lattice Boltzmann method with a more flexible grid that is free of interpolation.

Inspired by the work of Koelman [15], Bouzidi et al. [16] made the first attempt to construct a multiple-relaxation-time (MRT) LBM on a two-dimensional rectangular grid. The model showed good performance with appropriate choice of model parameters, but the resulting hydrodynamic momentum equation is not fully consistent with the Navier–Stokes equations. Another attempt was made by Zhou [17] who redefined the moments so that the transformation matrix for a rectangular grid was identical to that for a standard MRT on a square grid. However, the modifications suggested in Zhou [17] led to anisotropic fluid viscosity. Hegele et al. [18] indicated some extra degrees of freedom should be employed to satisfy the isotropy conditions for rectangular lattice Boltzmann scheme. There are three possible approaches: decoupling the discretizations of the velocity space from spatial and temporal discretization, modification of a collision operator with additional parameter, and adoption of more discrete microscopic velocities. They introduced two extra microscopic velocities to extend the D2Q9 model with BGK collision operator and were able to restore, on a rectangular grid, the isotropy condition required for the Navier–Stokes equation. They also suggested that four new velocities are needed in order to correctly extend the D3Q19 model onto a noncubic 3D grid.

In the present work, we explore the possibility to restore the isotropy condition on a 2D rectangular grid without introducing any additional microscopic velocity. We take advantage of some of the flexibility within the MRT LBM scheme [19,20]. For this purpose, we will introduce an additional parameter in the energy–normal stress moment subspace. Before presenting our novel MRT LBM scheme on a rectangular grid, previous MRT schemes on a rectangular grid are firstly reviewed in Section 2. An inverse design analysis will be used to derive the equilibrium moments and the anisotropy of the transport coefficients for a rectangular grid is revealed. In Section 3, our new scheme with an additional free parameter is constructed in order to restore the isotropy for a rectangular grid, namely, the usual Navier–Stokes hydrodynamic equations are derived using a rectangular grid. Also, the coupling relationships between relaxations times and determination of computational parameters are re-interpreted, which enables the flexibility to choose computational parameters according to different flow problems. In Section 4, numerical validation of our new scheme is provided using a 2D channel flow, a 2D lid-driven cavity flow and 2D Taylor–Green vortex flow. Concluding remarks are provided in Section 5.

2. An analysis of previous MRT LBM schemes on a rectangular grid

2.1. *The model of Bouzidi et al. [16]*

We begin with the MRT LBM scheme [19] in 2D, which has been shown to provide more flexibility in relaxing different moments and to significantly improve computational stability and accuracy, while simplicity and computational efficiency of LBM are retained. However, we consider a rectangular grid as shown in Fig. 1, where the non-zero lattice velocity in the x direction is one, and in the y direction is a ($a < 1$). Following the spirit of the D2Q9 model but with the different velocities in x and y directions, the discrete velocities are defined as

$$\mathbf{e}_i = \begin{cases} (0, 0) & i = 0 \\ (\pm 1, 0), (0, \pm a) & i = 1 - 4 \\ (\pm 1, \pm a) & i = 5 - 8. \end{cases} \tag{1}$$

The distribution functions in MRT LBM evolve as

$$f_i(\mathbf{x} + \mathbf{e}_i \delta t) - f_i(\mathbf{x}, t) = \Omega_i \tag{2}$$

where f_i is the distribution function associated with the molecular velocity $\mathbf{e}_i$ at position $\mathbf{x}$ and time t, Ω_i is the collision operator.

In MRT LBM, the streaming process takes place in the physical space while the collision process is performed in the moment space. The nine distribution functions define nine degrees of freedom, which implies that nine independent moments