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Fully discrete error estimation for a quasi-Newtonian fluid–structure interaction problem

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Dedicated to Max Gunzburger on the occasion of his 70th birthday

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ABSTRACT

We consider an incompressible quasi-Newtonian fluid–structure interaction (FSI) problem formulated in a monolithic framework, where the matching conditions at the moving interface are satisfied. The fully discretized FSI system is discussed with detailed analysis for the stability and error estimate as well as numerical experiments that confirm the theoretical results.

1. Introduction

Fluid–structure interaction (FSI) problems have various applications in engineering and biology, where two dynamics, fluid flow and deformable structure, are considered in one system. Simulations of blood flow, tidal current turbines and gas explosions in pipelines are some well-known applications in this research area [1–6]. FSI problems concern the mutual influence between two dynamics: the domain of fluid is determined by structure deformation, and structure movement is determined by fluid stress.

The numerical discretization of an FSI system poses great computational challenges due to the nature of its complexity. A fully-coupled scheme, which solves the fluid and structure subproblems simultaneously, results in a large system, which in turn requires large memory storage and a special solver. However, the monolithic approach has been used widely, in particular for blood flow problems, where a stability issue caused by *the added-mass effect* exists in many partitioned algorithms [7–11].

Recently, we used a monolithic approach to investigate the finite element approximation of a quasi-Newtonian FSI problem [12]. The fluid is quasi-Newtonian, where the fluid viscosity is a function of the magnitude of the deformation tensor and the fluid does not have any memory or elastic properties. Examples of such fluids include blood, lubricants, and paints. Numerical studies on FSI involving this type of fluids are found in [13,4,14]. The Arbitrary Lagrangian Eulerian method (ALE) was used to deal with the time-dependent domain of the fluid. In the ALE method, an invertible and sufficiently regular ALE mapping is introduced to obtain a conforming mesh at arbitrary time following the interface movement as the image of a fixed mesh in the reference domain. We analyzed a semi-discrete FSI system written in the ALE frame for stability, proved an error estimate and performed numerical tests. Results have shown the standard optimal convergence rate of a finite element solution.

We are aware that there are some error analysis results in the literature but those were derived using either a fixed interface assumption [15,16] or simplified equations [17]. Although there is plenty of existing work concerning the

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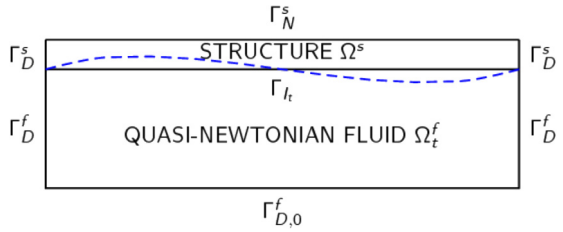

Fig. 1. Fluid–structure interaction domain.

monolithic formulations of FSI problems, we found that the full error analysis is still missing, especially in the setting of ALE formulation where all the Jacobians need to be introduced in order to handle the moving interface.

In this work we extend the analysis in [12] to a fully discretized FSI system by rigorously proving the approximation error and stability due to time discretization. The two dynamics in the FSI system are described in different settings: the fluid equation is given in a time-dependent Eulerian frame while the isotropic linear elastic structure is described in a time-independent Lagrangian frame. However, the interface conditions based on the continuity allow us to obtain a monolithic global formulation for the FSI problem, accounting for the fluid and the structure at the same time.

The paper is organized as follows. Section 2 introduces the coupled fluid–structure system with initial and boundary conditions. The matching conditions for the two dynamics on the interface are also provided in this section. In Section 3, we provide a monolithic weak formulation in the ALE framework. The time discretized system and corresponding analysis are presented in Section 4. The last section presents the numerical experiments that confirm the theoretical result given in Section 4.

2. Model description

Let Ω^f_t be the moving fluid domain at t in $\mathbb{R}^d (d = 2, 3)$ with the boundary $\Gamma^f_t := \Gamma^f_{D,0} \cup \Gamma^f_D \cup \Gamma_{l_t}$, where Γ_{l_t} is the moving interface boundary. Let Ω^s be a fixed domain for the structure described in terms of a Lagrangian frame of reference. The boundary of the structure is denoted as $\Gamma^s_t := \Gamma^s_N \cup \Gamma^s_D \cup \Gamma_{l_t}$. We consider the system of a quasi-Newtonian flow and an isotropic linear elastic structure.

$$\rho_f \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) - \nabla \cdot \nu_f (|D(\mathbf{u})|) D(\mathbf{u}) + \nabla p = \mathbf{f}_f \quad \text{in } \Omega^f_t, \tag{2.1}$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega^f_t, \tag{2.2}$$

$$\rho_s \frac{\partial^2 \boldsymbol{\eta}}{\partial t^2} - 2\nu_s \nabla \cdot D(\boldsymbol{\eta}) - \lambda \nabla (\nabla \cdot \boldsymbol{\eta}) = \mathbf{f}_s \quad \text{in } \Omega^s, \tag{2.3}$$

where $\mathbf{u}$ denotes the velocity vector, p the pressure of fluid, η the displacement of the structure, ρ_f and ρ_s are the densities of the fluid and the structure, respectively. In (2.1) and (2.3), $D(\mathbf{u}) := (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$ is the rate of the strain tensor, and $\mathbf{f}_f$ and $\mathbf{f}_s$ are the body forces. ν_s and λ are the Lamé parameters defined as

$$\nu_s = \frac{E}{2(1+\nu)}, \qquad \lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}, \tag{2.4}$$

where E is the Young's Modulus of the structure and ν is its Poisson ratio.

Initial and boundary conditions for $\mathbf{u}$ and $\boldsymbol{\eta}$ are given as follows:

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0 \quad \text{in } \Omega^f_0, \tag{2.5}$$

$$\boldsymbol{\eta}(\mathbf{x}, 0) = \boldsymbol{\eta}_0, \quad \boldsymbol{\eta}_t(\mathbf{x}, 0) = \dot{\boldsymbol{\eta}}_0 \quad \text{in } \Omega^s, \tag{2.6}$$

$$\mathbf{u} = \mathbf{u}_D \quad \text{on } \Gamma^f_D, \tag{2.7}$$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma^f_{D,0}, \tag{2.8}$$

$$2\nu_s D(\boldsymbol{\eta}) \mathbf{n}_s + \lambda (\nabla \cdot \boldsymbol{\eta}) \mathbf{n}_s = \mathbf{0} \quad \text{on } \Gamma^s_N, \tag{2.9}$$

$$\boldsymbol{\eta} = \mathbf{0} \quad \text{on } \Gamma^s_D, \tag{2.10}$$

where $\mathbf{n}_f$ and $\mathbf{n}_s$ are the outward unit normal vectors to Ω^f_t and Ω^s, respectively. The moving interface Γ_{l_t} is determined by the displacement $\boldsymbol{\eta}$ at time t (Fig. 1). To simplify numerical analysis we use $\mathbf{u}_D = 0$ on Γ^f_D, but all our results hold for the case of $\mathbf{u}_D \neq 0$ by the standard technique [18].