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RESEARCH PAPER

General method of calculating annular laminar pressure drop of drilling fluids with different rheological models

PENG Qi*, FAN Honghai, ZHOU Haobo, LI Chaowei, CHEN Xuyue, WANG Echuan, YE Zhi

Key Laboratory for Petroleum Engineering of Ministry of Education, China University of Petroleum, Beijing 102249, China

Abstract: Traditional annular laminar flow analysis method applied in drilling engineering has poor accuracy and does not apply to complicated rheological models. A general method of calculating annular laminar pressure drop of drilling fluids with different rheological models was proposed, compared with traditional method, and verified using experiment data in published articles. Based on annular slot flow model, the pipe flow equation was popularized in annular and an annular flow equation was established, and the relationship between annular flow and shear stress at pipe wall was established by combining annular flow equation and fluid's rheological equation. If annular flow is given, shear stress at pipe wall can be got, and then the annular pressure drop is also available. Compared with traditional methods, the general method is adapt to any rheological model of annular laminar flow, and has good universality, simple modeling process and high accuracy. The calculation results based on experiment data in published articles show: no matter the flow rate is high or low, the results of the general method match the measured results well, which makes up the error of traditional method.

Key words: annular laminar pressure drop; annular flow equation; non-newtonian fluid; rheological equation; traditional method; general method; drilling fluids

Introduction

Calculation of annular pressure loss of non-Newtonian fluids is an important part of drilling fluid mechanics, involving drilling, well killing, well cementation and well completion etc. As modern drilling engineering pushes toward deep formation and deep sea, strata conditions encountered and drilling fluid systems used are becoming more and more complex, thus traditional drilling fluid rheological models (Bingham, Power-Law, Casson) [1–4] can not satisfy the engineering need any more. Some more practical models are used to describe drilling fluid rheological properties, such as three-parameter model (Herschel-Buckley, Robertson-Stiff, Sisko models etc.) [5–8] and four-parameter model [9]. The introduction of these complex rheological models improves description accuracy of drilling rheology, but also increases the difficulty of hydraulic calculation at the same time. Based on annular slot flow model, this paper proposes a calculation algorithm of fluid annulus laminar flow pressure loss that is suitable for different rheological models, and gives some annular pressure loss calculation equations for rheological models.

1 Traditional calculation algorithm for annular laminar flow pressure loss

The majority of annular drilling fluid is non-Newtonian. Based on the annular slot flow model (Fig. 1), the main procedures of traditional calculation algorithm for annular laminar flow pressure loss are as follows.

(1) Assuming that annular drilling fluid meets the following conditions: viscous laminar flow (structural flow); steady uniform flow; no slide along the pipe wall and negligible distribution difference of fluid velocity inside and outside the wall of the annulus. Under such assumption, according to the equilibrium relationship between annular pressure loss Δp and shear stress τ of fluid, an annular uniform laminar flow control equation was established as:

$$\tau=\frac{\Delta pr}{L} \tag{1}$$

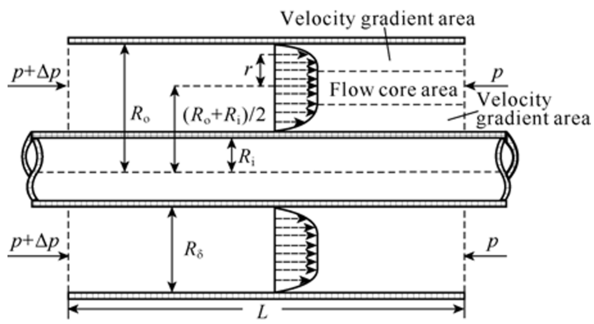

Fig. 1 Annular slot flow model

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* **Corresponding author.** E-mail: lao.peng@163.com
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At the pipe wall ($r=R_\delta/2$):

$$\tau_w = \frac{\Delta p R_\delta}{2L} \tag{2}$$

(2) Combining rheological equation of given fluid and annular uniform laminar flow control equation leads to the velocity differential equation of non-Newtonian annular laminar flow. Taking the boundary conditions into consideration, the annular laminar flow velocity distribution equation $u=u(\Delta p)$ will be obtained after integration.

(3) Doing area integration with velocity distribution equation $u=u(\Delta p)$ over the annulus flow cross section, and omiting the corresponding higher order terms according to some rheological models, we can get explicit expression of annular flow $Q=Q(\Delta p)$, and then get the average velocity equation $v=v(\Delta p)$.

(4) Solve the inverse function of the annular velocity equation $v=v(\Delta p)$, and the annular pressure loss calculation equation $\Delta p=v^{-1}(v)$ will be gotten.

Table 1 shows laminar flow annulus pressure loss calculation models of some common rheological patterns obtained by traditional algorithm. In order to get explicit expression of calculation equation, many rheological models have been correspondingly simplified in the derivation process and thus lead to errors in final results. In some cases, this error may be significant. Therefore, it's necessary to find a more accuracy laminar flow annulus pressure loss calculation algorithm which can be applied to different rheological models.

Table 1 Laminar flow annulus pressure loss calculation model by traditional algorithm

Rheological models	Rheological equation	Calculation model of annular laminar pressure loss
Bingham	$\tau=\tau_0+\mu_p\gamma$	$\Delta p=\frac{48\mu_p Lv}{D_\delta^2}+\frac{6L}{D_\delta}\tau_0$
Power-Law	$\tau=K\gamma^n$	$\Delta p=\frac{4KL}{D_\delta}\left[\frac{12v(2n+1)}{3nD_\delta}\right]^n$
Casson	$\tau^{0.5}=\tau_0^{0.5}+\eta_\infty^{0.5}\gamma^{0.5}$	$\Delta p=\frac{4L}{D_\delta}\left[\left(\frac{12\eta_\infty v}{D_\delta}-\frac{3}{50}\tau_0\right)^{0.5}+\frac{6}{5}\tau_0^{0.5}\right]^2$
Herschel-Bulkley	$\tau=\tau_0+K\gamma^n$	$\Delta p=\frac{4KL}{D_\delta}\left[\frac{12v(2n+1)}{3nD_\delta}\right]^n+\frac{4L\tau_0(2n+1)}{D_\delta(n+1)}$
Robertson-Stiff	$\tau=A(\gamma+C)^B$	$\Delta p=\frac{4AL}{D_\delta}\left[\frac{2B+1}{3B}\left(\frac{12v}{D_\delta}+\frac{3}{2}C\right)\right]^B$

2 General calculation algorithm for annular laminar pressure loss

2.1 General annular flow equation

In the annulus flow section of Fig. 1, two thin rings whose thickness is dr are taken: one of the thin ring's radius is $(R_o+R_i)/2+r$, the other is $(R_o+R_i)/2-r$. The flow between the two rings is:

$$\begin{aligned} dQ &= 2\pi u\left[(R_o+R_i)/2+r\right]dr+2\pi u\left[(R_o+R_i)/2-r\right]dr \\ &= 2\pi u(R_o+R_i)dr \end{aligned} \tag{3}$$

So the total flow of annular is:

$$Q=\int_0^{\frac{R_\delta}{2}} 2\pi u(R_o+R_i)dr = 2\pi(R_o+R_i)\left[ur\Big|_0^{\frac{R_\delta}{2}}+\int_0^{\frac{R_\delta}{2}} r\left(-\frac{du}{dr}\right)dr\right] \tag{4}$$

At the pipe wall, $u\big|_{r=R_\delta/2}=0$, so in Equation (4), $ur\big|_0^{\frac{R_\delta}{2}}=0$. And $\gamma=-\frac{du}{dr}=f(\tau)$, $r=\frac{\tau R_\delta}{2\tau_w}$ and $dr=\frac{R_\delta}{2\tau_w}d\tau$ are taken into Equation (4):

$$Q=\frac{\pi(R_o+R_i)R_\delta^2}{2\tau_w^2}\int_0^{\tau_w}\tau f(\tau)d\tau \tag{5}$$

Equation (5) is the general Equation describing the relationship between annular flow and shear stress at pipe wall, called the annular flow Equation. Equation (5) can apply to any no time-varying non-Newtonian fluids (apparent viscosity is not related to shear time), and is the extended application of pipe flow equation [10–11] derivation idea in the annulus laminar flow.

For annular structural flow (exist flow core) of viscoplastic fluid, Equation (5) can be written as follows:

$$Q=\frac{\pi(R_o+R_i)R_\delta^2}{2\tau_w^2}\int_0^{\tau_0}\tau f(\tau)d\tau+\frac{\pi(R_o+R_i)R_\delta^2}{2\tau_w^2}\int_{\tau_0}^{\tau_w}\tau f(\tau)d\tau \tag{6}$$

Considering the flow core zone shear rate $f(\tau)=0$, therefore $\int_0^{\tau_0}\tau f(\tau)d\tau=0$, and Equation (6) can be simplified as:

$$Q=\frac{\pi(R_o+R_i)R_\delta^2}{2\tau_w^2}\int_{\tau_0}^{\tau_w}\tau f(\tau)d\tau \tag{7}$$

2.2 Solution of the annular laminar flow pressure loss

It can be seen from Equation (5) and Equation (7) that, if fluid type is known, then the fluid rheological model will be known, the relation between the rheological model of annulus laminar flow Q and the pipe wall shear stress τ_w can be obtained through integral. Once annular flow Q is given, shear stress at pipe wall τ_w can be gotten, and then annular pressure loss Δp through Equation (2). This is the basic idea to solve annular pressure loss by annular flow equation. Taking Robertson-Stiff and Herschel-Bulkley model as examples, the calculation process will be explained in the following section.

Herschel-Bulkley model fluid has a yield value, so the rheological equation $\tau=\tau_0+K\gamma^n$ is substituted into Equation (7):