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Asymptotic behavior and multiplicity for a diffusive Leslie–Gower predator–prey system with Crowley–Martin functional response☆

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ABSTRACT

A diffusive predator–prey system with modified Leslie–Gower and Crowley–Martin functional response is considered. The extinction and permanence of the time-dependent system are determined by virtue of the comparison principle. Then, the sufficient and necessary conditions for the existence of coexistence states are obtained. Furthermore, the stability, uniqueness and exact multiplicity of coexistence states are investigated by means of the combination of the perturbation theory, bifurcation theory and degree theory. Our results indicate that c_1 have an effect on the stability and exact multiplicity of coexistence states. Finally, some numerical simulations are presented to verify and complement the theoretical results.

1. Introduction

One of the key themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey. Therefore, the predator–prey systems with various functional responses and different boundary conditions have been proposed and studied by many ecologists and mathematicians. These studies applied classic types of functional responses including: Lotka–Volterra type [1,2], Holling type [3–7], Beddington–DeAngelis type [8–12], ratio-dependent type [13–16] and so on.

In [17], Hsu and Huang proposed the following predator–prey system:

$$\begin{cases} u_t = a_1 u\left(1-\dfrac{u}{k}\right) - P(u)v, & t>0,\\ v_t = v\left[a_2\left(1-\dfrac{hv}{u}\right)\right], & t>0, \end{cases} \tag{1.1}$$

where u and v represent the densities of prey and predator respectively. The parameters a_1, a_2, k and h are positive constants. The term hv/u is called the Leslie–Gower term. It measures the loss in predator population thanks to the rarity of its favorite

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food u. Obviously, the Leslie–Gower term in (1.1) may be unbounded and lack some good properties of classical functions. Consequently, in order to model reasonable dynamics, a positive constant is added to the denominator of hv/u. On the other hand, taking into account the inhomogeneous distribution of the predators and prey in different spatial locations at any given time, system (1.1) is reduced to

$$\begin{cases} u_t - \Delta u = a_1 u\left(1 - \dfrac{u}{k}\right) - P(u)v, & x \in \Omega,\ t > 0,\\ v_t - \Delta v = v\left(a_2 - \dfrac{b_2 v}{u + c_2}\right), & x \in \Omega,\ t > 0, \end{cases} \tag{1.2}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$.

If $P(u)$ is Holling type II functional response, then system (1.2) becomes

$$\begin{cases} u_t - \Delta u = a_1 u\left(1 - \dfrac{u}{k}\right) - \dfrac{b_1 uv}{u + c_1}, & x \in \Omega,\ t > 0,\\ v_t - \Delta v = v\left(a_2 - \dfrac{b_2 v}{u + c_2}\right), & x \in \Omega,\ t > 0. \end{cases} \tag{1.3}$$

There are many scholars investigated system (1.3). Peng and Wang [18], Zhou [19] and Zhou and Shi [20] considered steady-state system of (1.4) under homogeneous Dirichlet boundary conditions. In [18], the authors investigated the case $\frac{a_1}{k} = 1,\ b_1 = c_1 = \frac{1}{m},\ b_2 = c_2 = m$ and get the stability and multiplicity of coexistence solutions when m is large. The existence, stability, multiplicity and uniqueness of positive solutions are obtained in [19,20] by using the local bifurcation theory. As far as (1.3) with homogeneous Neumann boundary conditions is concerned, in [21,22], the authors obtained the existence and non-existence of non-constant positive solutions. For the details, please refer to these references.

The Holling type II functional response $P(u) = b_1 u/(u + c_1)$ in (1.3) is classified as one of prey-dependent functional response, it assumes that there is no interference between predators. To overcome this limitation, Crowley and Martin [23] proposed the following functional response

$$f(u, v) = bu/(1 + cu)(1 + dv),$$

which is called the Crowley–Martin type functional response. It is similar to the well-known Beddington–DeAngelis functional response but has an additional term modeling mutual interference among predators. In addition, Crowley–Martin type functional response allows for interference among predators regardless of whether an individual predator is currently handling prey or searching for prey. Therefore, the predator–prey model with Crowley–Martin type functional response progresses the Holling–Tanner model and Beddington–DeAngelis model. As far as we know, there are not many works on the diffusive predator–prey systems with Crowley–Martin functional response. For more information about the background and applications of the Crowley–Martin type functional response, one may refer to [24–27]. Particularly, in [24,25,27], by virtue of local bifurcation theory, the authors discussed the multiplicity of positive solutions when parameter lies in a sufficiently small range.

Considered the above analysis and compared with [24,25,27], we deal with the following predator–prey model under homogeneous Dirichlet boundary conditions:

$$\begin{cases} u_t - \Delta u = a_1 u\left(1 - \dfrac{u}{k}\right) - \dfrac{b_1 uv}{(1 + c_1 u)(1 + dv)}, & x \in \Omega,\ t > 0,\\ v_t - \Delta v = v\left(a_2 - \dfrac{b_2 v}{u + c_2}\right), & x \in \Omega,\ t > 0,\\ u = v = 0, & x \in \partial\Omega,\ t > 0,\\ u(x, 0) = u_0(x) \ge 0, \not\equiv 0, \qquad v(x, 0) = v_0(x) \ge 0, \not\equiv 0, & x \in \Omega, \end{cases} \tag{1.4}$$

where a_i $(i = 1, 2)$ are the intrinsic growth rate of prey u and predator v, respectively, k represents the coefficient of the carrying capacity, b_1, c_1 and d describe the effects of capture rate, handling time and the magnitude of interference among predators, respectively, on the feeding rate, b_2 is the maximum value which per capita reduction rate of v can attain, c_2 measures the extent to which environment provides protection to predator v. $u_0(x)$ and $v_0(x)$ are continuous functions. All the parameters of (1.4) are positive constants. The corresponding ODE system of (1.4) was proposed and studied in [28]. The asymptotic behavior of the positive equilibrium and the existence of Hopf-bifurcation of non-constant periodic solutions surrounding the interior equilibrium are considered.

When $d = 0,\ c_2 = 0$, system (1.4) becomes the general Holling–Tanner predator–prey system. Under homogeneous Neumann boundary conditions, Peng and Wang [29,30] investigated the existence and non-existence of non-constant positive solutions and the local and global stability of the unique positive equilibrium. When $d = 0,\ c_1 = 0,\ c_2 = 0$, system (1.4) reduces to Leslie–Gower model, one may see [31,32].