Computers & Operations Research 71 (2016) 1–15

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

A global shooting algorithm for the facility location and capacity acquisition problem on a line with dense demand

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ARTICLE INFO

Available online 12 January 2016

Keywords:
Location-allocation problem
Capacity acquisition
Dynamic programming
Shooting algorithm

ABSTRACT

This paper describes the development of an exact allocation-based solution algorithm for the facility location and capacity acquisition problem (LCAP) on a line with dense demand data. Initially, the n-facility problem on a line is studied and formulated as a dynamic programming model in the allocation decision space. Next, we cast this dynamic programming formulation as a two-point boundary value problem and provide conditions for the existence and uniqueness of solutions. We derive sufficient conditions for non-empty service regions and necessary conditions for interior facility locations. We develop an efficient exact shooting algorithm to solve the problem as an initial value problem and illustrate on an example. A computational study is conducted to study the effect of demand density and other problem parameters on the solutions.

1. Introduction

The location-allocation problem (LAP) with minisum objective is to simultaneously determine the locations of n facilities and allocate m demand points to these facilities in such a way that total of distance between demand points and the facilities is minimized. This problem and its variants arise in many practical settings such as retail store location in marketing, warehouse location in logistics, and clustering problems. The line version of this problem has practical applications in locating non-mobile supply, storage, or servicing facilities along transportation channels such as highways, railways or rivers. In this paper, we consider a generalized version of the LAP, called Location, Capacity acquisition and Allocation Problem (LCAP), which, in addition to the fixed cost and distribution costs, accounts for capacity acquisition costs. We present an alternative modeling and efficient exact search procedure for the n-facility LCAP on the line with continuous demand density. We assume that the demand at each customer location is assigned to the closest facility.

Most of the location literature applies to problems having a discrete set of demand points. However, modeling demand as a continuous density function may be preferable, as in the case of dense demand data sets or when the demand volume and locations are uncertain. A common approach to solving the LAP is to prioritize locations over allocation decisions and assume allocation decisions are decided subsequent to location decisions [42,43]. In contrast, we formulate and solve the LCAP by prioritizing the allocation decisions. This approach allows us to cast the LCAP as a discrete optimal control problem of a non-linear system. Accordingly, we formulate the allocation problem as a boundary value problem. In our solution approach, we use an efficient shooting algorithm which reduces the problem to an initial value problem. Since the shooting algorithm finds only locally optimum solutions, we develop a derivative-free and efficient global optimization procedure to find all roots of the boundary value problem.

Our contribution is threefold. First, we introduce the continuous demand LCAP on the line which generalizes the earlier LAPs on the line. Second, we demonstrate that the allocation based modeling of the continuous demand LCAP on a line lends itself to a discrete optimal control problem formulation. This formulation differs from the earlier dynamic programming approaches for LAP because controls are now the allocation decisions. Using Pontryagin's minimum principle, we provide conditions on the continuous demand density for the existence and uniqueness of solutions. We also provide sufficient conditions for non-empty service regions and necessary conditions for interior facility locations. Our third contribution is the development of an efficient global shooting algorithm to solve the problem as an initial value problem, which, in essence, is a single-dimensional all roots finding problem.

The remainder of this paper is organized as follows. We briefly discuss the relevant literature in Section 2. In Section 3, we provide a dynamic programming formulation of the problem and then

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http://dx.doi.org/10.1016/j.cor.2015.12.018
0305-0548/

characterize optimal solutions. An efficient exact shooting algorithm together with an illustrative implementation is presented in Section 4. In Section 5, we present and discuss the results of a computational study to investigate the effect of problem parameters on the solution. Conclusions and a discussion of extensions follow in Section 6.

2. Relevant literature

The line version of LAP has practical applications in locating non-mobile supply, storage, or servicing facilities along transportation channels such as highways, railways or rivers. For example, Converse [1] consider the location of sewage treatment facilities on a river basin and Paroush and Tapiero [2] study the problem of locating a polluting plant on a line along which a population is distributed. Another application area is the location of mobile service units on a line. Carrizosa et al. [3,4] showed that fixing the location of mobile servers at the optimal location of non-mobile servers minimizes the average distance to demand. Consequently, the problem of locating mobile servers on a line reduces to that of non-mobile servers. Examples of applications are the location of service units along an oil pipeline, and the location of patrol cars or emergency vehicles in a highway where the demand is distributed over a path [5–7]. Bortnikov et al. [51] study the load-balancing problem in computer networks. They consider the load-distance balancing problem on a line segment where the goal is to minimize average delay incurred by a client. Other practical applications include the optimal positioning of an idle warehouse carousel and disk read/write heads [8–12].

While the majority of location models assume that customer demand is concentrated at discrete locations (e.g., centroids of postal codes), there are instances where continuously distributed demand is suitable. For instance, when the error associated with the aggregation of dense demand is high or the information on demand locations is uncertain. In such cases, customers are assumed to be continuously distributed and customer demand is modeled as a continuous density function. The demand density function is obtained in one of two ways, depending on data availability. When demand data (or a substitute) are not available, the analyst assumes a continuous distribution over the market region. When data are available in aggregate form or as a sample, then spatial smoothing techniques such as kernel density estimation or interpolation methods are applied. We refer the readers to Sliwinski [25] and Donthu and Rust [26] for the smoothing of discrete spatial data in market area applications.

The LCAP was first introduced by Verter and Dincer [23] with discrete demand and facility locations on the plane. Dasci and Verter [24] have adapted this formulation to planar location problems with continuous demand. They have proposed a continuous approximation based modeling framework and analytical solution approach to determine the size of the service regions. For the allocation solutions, they recommend using a heuristic method based on bivariate step function fitting. The proposed framework is applicable to problems with constant or slowly varying demand density. Both the discrete and planar versions of the LCAP differ from the classical LAP by their incorporation of capacity acquisition costs. Murat et al. [43] have considered the LCAP proposed in Dasci and Verter [24] with general demand density and have proposed a local search heuristic (a steepest descent algorithm) as the solution method. This method is applicable to problems where the allocation decisions are in the form of polygons, e.g. with Euclidean distances, fixed-charged linear capacity acquisition costs, and uniform unit capacity acquisition cost. Murat et al. [42] have extended the LCAP version of Murat et al. [43] to generalized distances and capacity acquisition costs. However, the proposed solution method is a local search method applicable only to two-facility problems. While Verter and Dincer [23] and Dasci and Verter [24] consider the number of facilities as a decision variable, we consider it (and hence that of service regions) as given in the LCAP, as in Murat et al. [42,43].

The one-dimensional LAP with discrete demand was first solved via dynamic programming by Love [13]. This algorithm has a complexity bound of $\mathcal{O}(nm^2)$. Our proposed solution approach is similar to the dynamic programming procedure of Love [13] in that we also prioritize the allocation decisions. The main difference is that the demand and allocation decisions are continuous in our modeling and solution approach. Using some developments in one-dimensional dynamic programming, Hassin and Tamir [14] have refined the complexity bound to $\mathcal{O}(nm)$. Denardo et al. [15] provide an interesting property for discrete LAP on a line with examples. This property, called interleaving property, stipulates that whenever a facility is removed from a solution, other facilities shift toward the location of the one that has been removed, but not farther toward it than the original location of the adjacent facility. Brimberg and ReVelle [16] have provided a linear programming based solution approach for this problem. The capacitated version of the discrete LAP on a line was studied by Tamir [17], Mirchandani et al. [18], Brimberg and Mehrez [19], and Eben-Chaime et al. [20]. The LAP with continuous demand on a line is very popular in the competitive equilibrium literature because of its simplicity. However, the focus of this literature is primarily the existence and characterization of equilibrium solutions rather than solution methods [21]. Suzuki et al. [22] have proposed an iterative descent-based solution approach for continuous LAP on a line. Drezner and Wesolowsky [52] studied the LAP with continuous demand on a line where the customers select their facility based on facility charges and transportation. In their formulation, each facility charges customers a fixed cost plus a variable cost inversely proportional to the number of customers patronizing the facility. They derive optimality conditions, state that there are multiple local solutions, and propose two efficient local search heuristic algorithms. Our work differs from this study in several aspects. First, we propose a continuous demand LCAP which is more general, e.g. location dependent fixed, capacity acquisition and distribution costs and general forms of (dis-)economies of scale. Second, our solution approach is exact and finds the globally optimum solution(s) whereas methods in Drezner and Wesolowsky [52] are heuristics. Hence, our approach is the first exact method for continuous LAP on a line and complements the exact approaches for discrete demand [13,16–20]. Our objective is to minimize the total cost to the manufacturer, whereas Drezner and Wesolowsky [52] minimize cost for customers. Lastly, we provide sufficient conditions for non-empty service segments and necessary conditions for interior facility locations.

3. Model

We study the LCAP problem of partitioning a market region on the line into service regions and determining their facility locations such that the total fixed cost, distribution costs, and capacity acquisition costs are minimized. We assume that the fixed costs are location dependent. Further, the capacities of the facilities are assumed to be unconstrained and can be increased at a cost. This assumption of soft capacity is not restrictive as long as the unit capacity acquisition cost function is kept general. A direct result of this assumption is that demand at each customer location is assigned to only one facility (single sourcing). The single sourcing assumption not only makes the analysis easier but is also the preferred method in practice.

We now provide some definitions and notation required for our model on the line. The market $\mathcal{M} \subseteq \mathbb{R}^+$ is defined as a bounded line segment with a length of M. Without loss of generality, we assume that $0 \in \mathcal{M}$. Every point $x \in \mathcal{M}$ has a demand density expressed as $D(x)$ which is assumed to be continuous. There are n facilities to be located and the location of facility i is represented by $x_i \in \mathcal{M}$. Each facility i serves the customer demand in a single