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Load-dependent and precedence-based models for pickup and delivery problems

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ABSTRACT

We address the one-to-one multi-commodity pickup and delivery traveling salesman problem (m-PDTSP) which is a generalization of the TSP and arises in several transportation and logistics applications. The objective is to find a minimum-cost directed Hamiltonian path which starts and ends at given depot nodes and such that the demand of each given commodity is transported from the associated source to its destination and the vehicle capacity is never exceeded. In contrast, the many-to-many one-commodity pickup and delivery traveling salesman problem (1-PDTSP) just considers a single commodity and each node can be a source or target for units of this commodity. We show that the m-PDTSP is equivalent to the 1-PDTSP with additional precedence constraints defined by the source–destination pairs for each commodity and explore several models based on this equivalence. In particular, we consider layered graph models for the capacity constraints and introduce new valid inequalities for the precedence relations. Especially for tightly capacitated instances with a large number of commodities our branch-and-cut algorithms outperform the existing approaches. For the uncapacitated m-PDTSP (which is known as the sequential ordering problem) we are able to solve to optimality several open instances from the TSPLIB and SOPLIB.

1. Introduction

In this paper we propose a new approach for the one-to-one multi-commodity pickup and delivery traveling salesman problem (m-PDTSP) introduced by Hernández-Pérez and Salazar-González [23]. The problem arises in several transportation and logistics applications. The m-PDTSP generalizes the well known travelling salesman problem (TSP) as well as two other variants which in turn also generalize the TSP. To better contextualize the m-PDTSP we will start by introducing briefly the other three variants, pointing out relations between the four problems as well as stating one of the main results of this paper.

We first consider the TSP [24], or more precisely the asymmetric version since all the problems discussed here are defined in a directed graph $G=(V,A)$. For each arc $(i,j)\in A$, a travel distance (or cost) c_{ij} of going from i to j is given. The objective is to find a minimum cost Hamiltonian tour. Many formulations have been presented for this problem (see, for instance [30], as probably the latest such reference) and we also refer the reader to the well known formulation by Dantzig et al. [9] (DFJ) that will be stated in Section 3 as a subformulation for all the formulations presented and discussed in this paper.

The first generalization we consider is the precedence constrained TSP (PCTSP) where a set K of pairs of nodes (s_k,d_k), $\forall k\in K$, is given as an input of the problem. In this variant we consider a special node, node 0 as a depot and where the tour starts and ends. As before, the objective is to find a minimum cost Hamiltonian circuit, but now we have the additional constraint that for each $k\in K$, node s_k must precede node d_k in the tour. We refer the reader to the papers by Balas et al. [6], Ascheuer et al. [5], and Gouveia and Pesneau [16]. Cut-like inequalities specific for the precedence case and generalizing the well known cut inequalities that arise in the DFJ formulation have been proposed in the first paper. These sets of constraints will be referred to in Section 4. We also observe that this problem is often defined as searching for a minimum cost Hamiltonian path between a source node 0 and destination node $n+1$. The two variants are obviously equivalent. Also, the PCTSP is known as the sequential ordering problem (SOP). From now on, we will keep the Hamiltonian

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path alternative for describing the subsequent variants including the problem studied in this paper.

A second variant of the TSP is the so-called many-to-many one-commodity pickup and delivery traveling salesman problem (1-PDTSP) and has been introduced by Hernández-Pérez and Salazar-González [20]. In this problem and as stated before, we consider a node set V with a start and an end depot 0 and $n+1$, respectively, and the set of customers $V_c = \{1, \ldots, n\}$. We also consider a vehicle of capacity Q and a single commodity, and each node can be a source or target for units of this commodity. Values $\rho_j,\ \forall j \in V$, represent the customer demands: nodes with $\rho_j > 0$ and $\rho_j < 0$ are denoted as pickup and delivery customers, respectively. Nodes with $\rho_j = 0$ also need to be visited without changing the vehicle load. Again, we want to find a Hamiltonian path from 0 to $n+1$ satisfying all customer demands and the given vehicle capacity Q. It is $\mathcal{NP}$-hard to find a feasible solution for the 1-PDTSP as shown by Hernández-Pérez and Salazar-González [20]. The papers by Hernández-Pérez and Salazar-González [21,22] present several models and valid inequalities for the 1-PDTSP and branch-and-cut algorithms to solve it. One of these models will be reviewed in Section 4. Clearly, if one ignores the vehicle capacity, the 1-PDTSP reduces to the TSP.

So far, we have described two variants that generalize the TSP. As mentioned before, in this paper we study a new approach for the m-PDTSP. This problem can also be viewed as a generalization of the SOP and the 1-PDTSP.

In the m-PDTSP there are m commodities $K = \{1, \ldots, m\}$, each $k \in K$ associated with a demand q_k, a source $s_k \in V/\{n+1\}$, and a destination $d_k \in V/\{0\}$. We assume $s_k \neq d_k$ and $q_k > 0$. A customer j can be the source of several commodities and the destination of other commodities. As in the 1-PDTSP we also consider a vehicle capacity $Q > 0$. We assume that $q_k \leq Q$ for all $k \in K$. The objective is to find a minimum cost Hamiltonian path between nodes 0 and $n+1$, such that (i) for each commodity $k \in K$ source s_k is visited before destination d_k, (ii) q_k units are transported from s_k to d_k, and (iii) the vehicle capacity is an upper bound of the vehicle load for each arc on the path from 0 to $n+1$.

As pointed out by Hernández-Pérez and Salazar-González [23] the m-PDTSP generalizes the 1-PDTSP. We simply aggregate the different flows into a single one. The customer demands of the equivalent 1-PDTSP are defined by the load changes when the vehicle visits a customer in the m-PDTSP. Again, and as also pointed out in Hernández-Pérez and Salazar-González [23], if one ignores the vehicle capacity in the m-PDTSP, one obtains the SOP since the precedence between source and destination for each commodity must be maintained.

The m-PDTSP is $\mathcal{NP}$-hard since it generalizes all the variants described here which are also known to be $\mathcal{NP}$-hard. Hernández-Pérez and Salazar-González [23] present two solution approaches, both based on Benders decomposition of a path and a multi-commodity flow model. The multi-commodity flow model will be revisited in Section 3. Their branch-and-cut algorithms are based on models in the natural variable space, i.e., only use binary variables for arcs A. These approaches usually achieve excellent results in terms of solution runtime for loosely constrained problem instances, i.e., when only a few commodities have to be considered or the given vehicle capacity is large in relation to the demands. In these cases only a few violated inequalities have to be added within the cutting plane phase. Additionally, the reduced size of the initial model makes it possible to quickly solve the corresponding linear programming (LP) relaxation. However, when considering problem instances with many commodities and/or a tight vehicle capacity several weaknesses of these approaches show up, namely that the basic model provides only a quite weak LP relaxation value leading to a large number of branch-and-bound nodes and making it necessary to add many violated inequalities. Rodríguez-Martín and Salazar-González [31] also propose several heuristic approaches for the m-PDTSP to obtain high-quality solutions for larger instances for which exact approaches cannot obtain satisfying results within reasonable time. They present a simple nearest neighbor heuristic to construct a solution followed by an improvement phase based on 2-opt, 3-opt, and restricted mixed integer programming neighborhood structures. We conclude this literature review by pointing to the overview on further pickup and delivery problems given in Berbeglia et al. [7].

The models in this paper are mostly based on a new result stating that the m-PDTSP is equivalent to the 1-PDTSP with additional precedence constraints defined by the origin–destination pairs for each commodity. That is, in a loose sense the m-PDTSP combines together the two previous variants. The advantage of using this relation to model the m-PDTSP is that we are able to model the capacity constraints just by considering a single commodity and this helps considerably in running times. The precedence relations are ensured separately by adding valid inequalities from the SOP, see Balas et al. [6] and Ascheuer et al. [5]. We also introduce new inequalities based on sequences and logical implications of precedence relations which are able to further close the LP gaps, especially for instances with a large number of precedence constraints.

Also, we present alternative ways to model the capacity constraints based on load-dependent layered graphs which improve the LP bounds for tight capacities. In particular we consider a formulation based on a 3-dimensional layered graph that combines position and load together and leads to tighter bounds at the cost of a larger sized model.

Our branch-and-cut algorithm to solve the m-PDTSP consists of several preprocessing methods, primal heuristics, and separation routines for the SOP inequalities. Especially for tightly capacitated instances with a large number of commodities we are able to outperform the approaches by Hernández-Pérez and Salazar-González [23]. In our experiments, we also consider the uncapacitated variant of the m-PDTSP, i.e., the SOP. Here, an adapted variant of our branch-and-cut algorithm is able to solve to optimality several open instances from the TSPLIB and SOPLIB.

The remainder of the paper is structured as follows: In Section 2 we present reduction and preprocessing techniques for the m-PDTSP, Section 3 revises existing models, Section 4 discusses the transformation to a single-commodity problem, Section 5 introduces layered graph models for the capacity constraints, Section 6 describes our branch-and-cut algorithms, Section 7 shows experimental results, and Section 8 concludes the paper.

2. Preprocessing

In this section we discuss some problem reductions and relevant problem properties which will be used to reduce and strengthen the models discussed in this paper. Additionally, these tests and properties may lead to an early detection of infeasibility of an instance.

2.1. Commodities

A commodity $k \in K$ is called transitive if there exist commodities $k_1, k_2 \in K/\{k\}$ with $s_{k_1} = s_k, d_{k_1} = s_{k_2}, d_{k_2} = d_k$. It can be easily seen that the set of feasible solutions is not modified if a transitive commodity is removed from set K and the demands of the corresponding commodities k_1 and k_2 are appropriately modified, i.e., $q'_{k_1} = q_{k_1} + q_k$ and $q'_{k_2} = q_{k_2} + q_k$. We perform this reduction step for all transitive commodities.

2.2. Precedence relations

The source–destination pairs $(s_k, d_k),\ \forall k \in K$, induce an acyclic precedence graph $P = (V, R)$ with R being the transitive closure of