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Probabilistic properties of fitness-based quasi-reflection in evolutionary algorithms

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ABSTRACT

Evolutionary algorithms (EAs) excel in optimizing systems with a large number of variables. Previous mathematical and empirical studies have shown that opposition-based algorithms can improve EA performance. We review existing opposition-based algorithms and introduce a new one. The proposed algorithm is named fitness-based quasi-reflection and employs the relative fitness of solution candidates to generate new individuals. We provide the probabilistic analysis to prove that among all the opposition-based methods that we investigate, fitness-based quasi-reflection has the highest probability of being closer to the solution of an optimization problem. We support our theoretical findings via Monte Carlo simulations and discuss the use of different reflection weights. We also demonstrate the benefits of fitness-based quasi-reflection on three state-of-the-art EAs that have competed at IEEE CEC competitions. The experimental results illustrate that fitness-based quasi-reflection enhances EA performance, particularly on problems with more challenging solution spaces. We found that competitive DE (CDE) which was ranked tenth in CEC 2013 competition benefited the most from opposition. CDE with fitness-based quasi-reflection improved on 21 out of the 28 problems in the CEC 2013 test suite and achieved 100% success rate on seven more problems than CDE.

1. Introduction

Evolutionary algorithms (EAs) are computational intelligence techniques that excel in optimizing systems with a large number of variables. EAs are effective numerical methods for finding global solutions to complex problems because they do not require a differentiable or a continuous objective function and can successfully escape local minima. The cost of these benefits is the increased convergence time due to the large number of function evaluations performed by the EAs.

This paper proposes a novel algorithm which generates perturbed solution candidates that have an increased probability (relative to other opposition-based methods) of being closer to the solution(s) of the optimization problem. Consequentially, EAs are expected to achieve lower costs results to optimization problems. We provide the mathematical proofs to demonstrate the benefits that can be gained by employing this algorithm as a component of other heuristic search techniques.

The proposed algorithm is inspired by opposition-based learning (OBL). A shortcoming of most EAs that are based on natural processes is that they are modeled after very slow processes. On the other hand, human society progresses at a much faster rate via "social revolutions." Hence, an EA based on such a model can accelerate the learning process. Tizhoosh maps this theory to machine learning and proposes to use opposite numbers instead of random mutations to quickly evolve the EA population [1].

OBL was first proposed in 2005 [2] and was applied to a popular reinforcement learning algorithm, Q-learning. It was concluded that opposition-based extension reduces the algorithm's convergence time [3]. Opposition can also accelerate learning in machine intelligence [1]. Preliminary results with anti-chromosomes for genetic algorithms, contrariness values for Q-learning and opposite weights for artificial neural networks illustrate the numerous possibilities for opposition in the field of artificial intelligence.

The benefits of OBL in solving global optimization problems were first published in [4]. In that paper, concepts of opposition-based initialization and generation jumping were proposed to improve the solution accuracy of differential evolution (DE) and were tested on a limited test set. The research was extended to empirical analysis on an extensive collection of benchmark functions and the experimental results illustrated that opposition-based DE outperforms fuzzy-adaptive DE and standard DE [5].

The application of OBL in evolutionary computation is not limited to DE. Other EAs have also benefited from the idea of opposition. OBL

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has been paired with biogeography-based optimization (BBO) to form oppositional BBO (OBBO) [6,7]. Variations of OBBO have been introduced in the literature [8,9].

Particle swarm optimization (PSO) is another EA that benefited from OBL. OBL helps PSO to enhance swarm diversity [10]. Different opposition-based PSO algorithms have been introduced [11] with velocity clamping [12] or with Cauchy mutation [13].

More recent opposition-based EAs include appending artificial bee colony algorithm [14] with opposition to form generalized opposition-based ABC [15]; CODEQ, a parameter-free algorithm that combines chaotic search, opposition-based learning, differential evolution and quantum mechanics for optimizing constrained problems [16]; and opposition-based gravitational search algorithm [17].

Applications of OBL in other fields of evolutionary computation are also continuing to expand. A survey on the state-of-the-art opposition research is presented in [18]. Most recent uses of OBL include a new framework for OBL and cooperative co-evolution to solve large-scale global optimization problems [19], multiobjective optimization problems based on decomposition [20] and mathematical analysis demonstrating an opposition-based EA's probability of converging to optimum for discrete-domain scheduling problems [21].

The remainder of this paper is structured as follows. Section 2 provides a historical background for opposition in different fields of science and philosophy. This section also discusses different opposition-based techniques as employed in continuous optimization and their statistical characteristics. Section 3 introduces fitness-based opposition and fitness-based quasi-reflection. We then present the probabilistic analysis of fitness-based quasi-reflection. Section 4 provides a discussion on various algorithm parameters. Section 5 analyzes the performance of the proposed method on CEC 2013 competition on real-parameter single objective optimization test suite and some of the competing algorithms. In Section 6, we draw conclusions and outline potential next steps for this research.

2. Background

2.1. Introduction to opposition

In this section, we discuss the definitions of opposition in various fields, and explain how it can be applied to optimization problems.

Opposition is encountered in different fields under different names. In Euclidean geometry it is called inverse geometry, in physics it is called the parity transformation, and in mathematics it is called reflection. All of these definitions involve isometric self-mapping of a function. Other examples include astronomy where planets that are 180° apart are considered to be opposing each other. Opposites also have a significant meaning in semantics as generalizations of antonyms. Where antonyms are limited to gradable terms, such as thin and thick, the term "opposite" can be applied to gradable, non-gradable and pseudo-opposite terms.

Akin to the use of opposition in semantics, OBL has evolved many variations in computational intelligence. These variations generate opposite samples in different intervals of the search space. In the next section, we provide an overview of the opposition-based types.

2.2. Variations of opposition-based algorithms

Different opposition-based algorithms have been introduced in the literature to accelerate EA convergence. This section presents an overview of selected OBL techniques. The definitions of these algorithms are given in Table 1 and a graphical representation is given in Fig. 1.

Table 1
Mathematical definitions of existing opposition-based algorithms where c is the center of the search interval $[a, b]$ and can be calculated as $(a+b)/2$, and $\text{rand}(\alpha, \beta)$ is a random number uniformly distributed between α and β. For any $\hat{x} \in [a, b]$, its opposite values are defined below.

Method	Definition
Opposition	$\hat{x}_o = a+b-\hat{x}$
Quasi-opposition	$\hat{x}_{qo} = \text{rand}(c, \hat{x}_o)$
Quasi-reflection	$\hat{x}_{qr} = \text{rand}(c, \hat{x})$
Center-based sampling	$\hat{x}_{cb} = \text{rand}(\hat{x}, \hat{x}_o)$

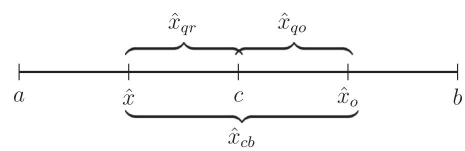

Fig. 1. Opposite points defined in domain $[a, b]$. c is the center of the domain and $\hat{x}$ is an arbitrary EA individual. $\hat{x}_o$ is the opposite of $\hat{x}$, and $\hat{x}_{qo}$ and $\hat{x}_{qr}$ are the quasi-opposite and quasi-reflected points, respectively. $\hat{x}_{cb}$ is the center-based point.

The original opposite point is proposed in [1] and is shown in Table 1 as $\hat{x}_o$. The central opposition theorem proves that the opposite point has a higher probability of being closer than a random guess to the solution [22]. However, central opposition theorem does not give the exact probabilities, but rather illustrates the probabilistic relationships between the opposite point and the solution, and a random point and the solution. An intuitive analysis can be used to show that the distance to the optimal solution is less with opposite sampling than random sampling [23]. This proof is also extended for N-dimensional search spaces. Other research performed empirical analysis of the performance of opposite points using 58 benchmark test functions [5]. The effects of population size, problem dimensionality and opposition jumping rate (J_r) on opposition-based differential evolution are studied [5].

A quasi-opposite point is randomly placed between the center of the search domain and the opposite point. The notation for a quasi-opposite point is given in Table 1 as $\hat{x}_{qo}$. Empirical studies on 30 benchmark functions indicate that quasi-oppositional optimization outperforms opposition [24]. Mathematical properties of quasi-opposition are given in [25]. We also computed the probability of $\hat{x}_{qo}$ being closer than the opposite point to the solution [7].

A newer OBL algorithm called quasi-reflection is denoted as $\hat{x}_{qr}$ in Table 1. A quasi-reflected point is placed randomly between the solution candidate and the center of the solution space. Empirical studies illustrate the performance gained by quasi-reflection [7,8]. Mathematical proofs given in [6] demonstrate that quasi-reflection has a higher probability of being closer to the solution than opposition and quasi-opposition.

One of the latest OBL variations is named center-based sampling and is denoted as $\hat{x}_{cb}$ in Table 1. The closeness of center-based candidates to solutions via Monte Carlo simulations for high dimensional problems is studied in [26]. Empirical studies given in [27–29] show the convergence speed gains that population-based algorithms achieve via this method. This algorithm is generalized in [30].

2.3. Probabilities of previously developed opposition-based algorithms

This section reviews the probabilities for two opposition-based algorithms as presented in the literature (Table 2). The results assume that the solution is uniformly distributed in the search space. Without any prior knowledge about the problem, assuming