Journal of Applied Geophysics 133 (2016) 82–91

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Interpretation of very low frequency electromagnetic measurements in terms of normalized current density over variable topography

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ARTICLE INFO

Article history:
Received 9 December 2015
Received in revised form 15 July 2016
Accepted 20 July 2016
Available online 25 July 2016

Keywords:
VLF electromagnetic
Subsurface imaging
Current density
Inversion
Topography

ABSTRACT

A 2D inversion approach is developed to interpret VLF electromagnetic measurement recorded over variable topography. To depict the variable topography accurately, an octree mesh discretization is incorporated. Subsurface structure is modeled in terms of apparent current density distribution and compared with the inversion results for actual resistivity distribution obtained using numerical techniques. The study demonstrates that the results obtained using both approaches (current density and resistivity distribution) are comparable, but due to analytical expression, current density imaging is faster. The conjugate gradient method is used to reduce the computation time and storage space when solving the matrix equations, resulting in feasible and practical imaging inversion of VLF data. The preconditioned matrix, which is determined by the distances between the blocks and observation points, has an important function in improving the resolution. In case of flat earth, preconditioned conjugate gradient inversion of data results in images that are comparable to those obtained using resistivity inversion. We also test whether topography variation in the order of skin depth is significant to incorporate topography in the modeling. The example of a topographical field VLF data inversion shows the efficacy of the presented approach to appraise the subsurface structure in terms of current density.

1. Introduction

The Very Low Frequency (VLF) electromagnetic method is a popular plane wave electromagnetic (EM) method to map shallow geological conductors on large scale due to its low cost and fast survey speed. The VLF method has been successfully used in solving a variety of geological problems in geothermal investigation (Baranwal and Sharma, 2006; Zlotnicki et al., 2006), geotechnical investigation (Sharma et al., 2010; Sungkono et al., 2014b), groundwater contamination and waste management studies (Poddar and Rathor, 1983; Monteiro Santos et al., 2006), archeological investigations (Drahor, 2006), mineral, groundwater and environmental investigations (Paterson and Ronka, 1971; Philips and Richards, 1975; Bernard and Valla, 1991; Benson et al., 1997; Tezkan, 1999; Sharma and Baranwal, 2005; Mohanty et al., 2011; Sundararajan et al., 2011; Sungkono et al., 2014a; Sungkono et al., 2015; Biswas and Sharma, 2016). The method has been used for land as well as airborne surveys (Arcone, 1978; Pedersen and Oskooi, 2004; Pedersen et al., 2009).

Different qualitative and quantitative approaches have been developed to interpret VLF electromagnetic data. Quantitative interpretation of VLF data has been developed to delineate 2D and 3D subsurface structures (Beamish, 1994, 1998, 2000; Sharma and Kaikkonen, 1998; Kaikkonen and Sharma, 1998; Baranwal et al., 2011; Kaikkonen et al., 2012; Kamm and Pedersen, 2014). The aforesaid interpretation techniques need a large amount of memory for the computing as well as a great deal of computation time. To speed up computation, analytical techniques were developed for imaging of the subsurface conductors. The filtering techniques developed by Fraser (1969) and Karous and Hjelt (1983) are frequently used to interpret VLF data qualitatively. Boukerbout et al. (2003) interpreted isolated conductors from VLF data using the wavelet approach. A more advanced filtering technique was discussed by Pedersen and Becken (2005) to construct corresponding equivalent current density distribution images, which would also provide quantitative information about the depth of the conductor. Recently, Singh and Sharma (2015) showed better results than obtained through the filtering techniques developed by Karous and Hjelt (1983) and Pedersen and Becken (2005). They developed a subsurface imaging technique using inversion strategy rather than an inverse filter. Singh and Sharma (2015) also revealed that the results get closer to those obtained by the aforementioned quantitative interpretation techniques. All the aforementioned analytical approaches have been developed for flat earth models.

In plane wave electromagnetic methods, small-scale rugged observation surfaces have little or no effect. However, a large-scale rugged

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http://dx.doi.org/10.1016/j.jappgeo.2016.07.018
0926-9851/

observation surface can affect the responses even at frequencies below 10 Hz (Chouteau and Bouchard, 1988; Jiracek et al., 1989; Wannamaker et al., 1986). Since the VLF transmitters operate in a high frequency range (between 5–30 kHz), topographical variation in the order of 10 m will also affect the response depending on the resistivity of the area and the frequency used (Arcone, 1978; Fischer, 1989; Karous, 1979; Liu and Becker, 1992). There are two regular approaches for the inversion to deal with topographical surfaces. In the first approach, topographical surfaces are considered a field distortion and corrections are made accordingly (Baker and Myers, 1980; Chouteau and Bouchard, 1988; Jiracek et al., 1989). In the second approach, topography is incorporated explicitly during the modeling and inversion (Baba and Chave, 2005; Key et al., 2006; Franke et al., 2007; Baranwal et al., 2011). To deal with the surface topography, rectangular discretization is commonly used. As shown in Fig. 1a, the near surface and air block is divided into two parts separated by the topographical surface. Such discretization produces large errors in describing rugged terrains. An octree-mesh generation divides the subsurface blocks into smaller blocks to depict the surface topography accurately (Fig. 1b). This technique has been used in electromagnetics (Ascher and Haber, 2001; Haber and Heldmann, 2007; Baranwal et al., 2011), large-scale earthquake modeling (Bielak et al., 2005), and airborne magnetic data interpretations (Davis and Li, 2013).

In the present study, an attempt is made to compute the vertical component of the magnetic field for a given current density distribution over topographic variations. Further, a 2D inversion approach incorporating the preconditioned conjugate gradient approach is developed for inversion of the VLF data directly for models including topography. Initially, the inversion approach was tested on a flat earth model and the efficacy of the preconditioner (determined by the distances between the blocks and the observation points) in improving the quality of the imaging technique is studied. The developed imaging technique is also compared with the actual resistivity distribution obtained using a rigorous resistivity inversion (Baranwal et al., 2011) on the flat earth model. Finally, the application of the approach is demonstrated using data measured over undulating topography (synthetic as well as real data) and compared with the results obtained using resistivity inversion.

2. Computational approach

In the VLF method, the ratio of the vertical component of the magnetic field to the horizontal component of the total field is measured. Further, this ratio is transformed into a real anomaly and expressed as a percentage (Smith and Ward, 1974).

$$\%\text{Real anomaly} = 100 \times \text{Re}\left(\frac{H_z}{H_x}\right) \tag{1}$$

where, $H_x = H_x^P + H_x^S$ (term H_x^P denotes primary and H_x^S denotes the secondary horizontal magnetic field) and H_z denotes the vertical magnetic field. Since the primary horizontal field is uniform in the study area, the measured real anomaly is proportional to the vertical component of the magnetic field. The vertical component of the magnetic field in the study area revealed the presence of 2D and 3D lateral conductivity variations as the homogeneous half-space and 1D variations in resistivity do not produce the vertical secondary magnetic field. Since $H_x^s < H_x^P$, it is an acceptable approximation to neglect the anomalous horizontal component of the magnetic field. H_x in Eq. (1), can be considered uniform in the study area. In the present work, H_x is considered as unity in order to equalize the relationship between the real anomaly and the vertical component of the magnetic field.

2.1. Computation of the vertical component of the magnetic field from 2D current density distribution

Using Biot-Savart law, Karous and Hjelt (1983) and Singh and Sharma (2015) computed the vertical component of the magnetic field for a given two-dimensional current density distribution. Karous and Hjelt (1983) assumed that a small block (*dxdz*) will have a uniform current density and derived the vertical component of the magnetic field. Further, they developed an efficient filter to calculate equivalent current density from the observed magnetic field. However, Singh and Sharma (2015) derived a frequency dependent analytical expression. Since topographic data contains both positive and negative z values, it is not acceptable to assume that current density decreases exponentially with depth. So, a uniform current density in a particular small block was considered (the same law of physics that has been considered in the Karous and Hjelt (1983) filtering approach) and an analytical solution was derived to compute the vertical component of the magnetic field along the profile for a given 2D rectangular block (Appendix 0).

Fig. 2 presents an octree-mesh discretization where smaller elements are used to depict the near surface zone; as depth increases the size of the elements increases accordingly. The observation points at the Earth's surface were located between −250 m and 250 m with an increment of 10 m. The plotted subsurface is divided into 466 cells. The cells are numbered from left to right and top to bottom from 1 to 466 (Fig. 2a). Further, for the model including topography, the observation points along the topographic surface were located between −500 m and 500 m with an increment of 25 m and similarly, the cells are numbered from left to right and top to bottom, correspondingly, from 1 to 249 (Fig. 2b) in the present study.

The vertical component of the magnetic field of rectangle ABCD at the observation point (0, 0) is expressed as (Fig. 3)

$$\Delta H_z^k = \frac{j_k}{4\pi}\left[\begin{array}{l} z_{k+1}\ln\left(\dfrac{x_{k+1}^2+z_{k+1}^2}{x_k^2+z_{k+1}^2}\right) - z_k\ln\left(\dfrac{x_{k+1}^2+z_k^2}{x_k^2+z_k^2}\right) \\ +2x_{k+1}\tan^{-1}\left(\dfrac{x_{k+1}(z_{k+1}-z_k)}{x_{k+1}^2+z_k z_{k+1}}\right) \\ -2x_k\tan^{-1}\left(\dfrac{x_k(z_{k+1}-z_k)}{x_k^2+z_k z_{k+1}}\right) \end{array}\right] \tag{2}$$

where, ΔH_z^k is the vertical magnetic field due to the kth rectangular

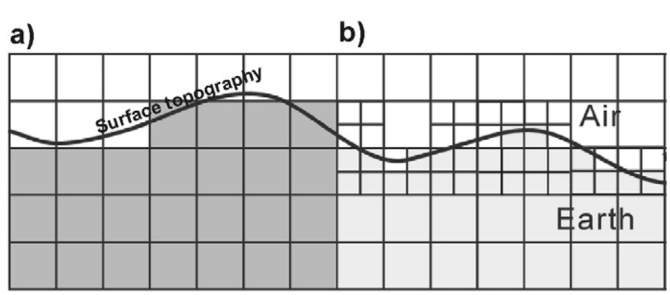

Fig. 1. Discretization approaches to deal with the surface topography. (a) Rectangular division and (b) octree-mesh division.