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2D spectral element modeling of GPR wave propagation in inhomogeneous media

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ABSTRACT

We present a spectral element method, for simulation of ground-penetrating radar (GPR) in two dimensions. The technique is based upon a weak formulation of the equations of Maxwell and combines the flexibility of the elemental-based methods with the accuracy of the spectral based methods. The wave field on the elements is discretized using high-degree Lagrange interpolation and integration over an element is accomplished based upon the Gauss–Lobatto–Legendre integration rule. As a result, the mass matrix and the damping matrix are always diagonal, which drastically reduces the computational cost. We first develop the formulation of 2D spectral element method (SEM) in the time-domain based on Maxwell's equations. The presented formulation is with matrix notation that simplifies the implementation of the relations in computer programs, especially in MATLAB application. We discuss the differences between spectral element method and finite-element method in the time-domain. Also, we show that the SEM numerical dispersion is much lower than FEM. To absorb waves at the edges of the modeling domain, we implement first order Clayton and Engquist absorbing boundary conditions (CE-ABC) introduced in numerical finite-difference modeling of seismic wave propagation. We used the SEM to simulate a complex model to show its abilities and limitations. As well as, one distinct advantage of SEM is that we can easily define our model features in nodal points, because the integration points and the interpolation points are similar that makes it very flexible in simulation of complex models.

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1. Introduction

Ground penetrating radar (GPR) is an electromagnetic (EM) geophysical exploration technique with extensive applications in near surface studies. Numerical modeling of radar waves plays a significant role in interpretation, processing and imaging of GPR data. On the other hand, simulation of GPR data brings us a better understanding of the radar wave interaction with subsurface structures.

There are several methods for the numerical modeling of GPR data, including ray-based methods (Goodman, 1994; Cai and McMechan, 1995), finite-difference methods (Irving and Knight, 2006), integral methods (Ellefsen, 1999), finite-element methods (Feng et al., 2012), element-free Galerkin method (Feng et al., 2015) and etc. Despite their difference in mathematical approaches, they all attempt to simulate the propagation of the GPR wave from the surface downwards with the emphasis on the interaction of the electromagnetic wave with the subsurface inhomogeneous media. Ray-based methods are fast modeling approaches which are based on high frequency approximations and cannot consider amplitude variations properly. However, in some applications such as imaging and FWI, we need solution of wave equation in complete form.

The most common approach for GPR modeling over the past decades is the finite-difference method (FDM). Although FDM is conceptually simpler and easier to program than the finite-element method (FEM), FEM is a more powerful and flexible numerical technique for handling problems involving complex geometries and inhomogeneous media (Matthew & Sadiku, 2001). The systematic generality of the FEM method permits constructing general-purpose computer programs for solving a wide range of numerical problems. Moreover, implementing boundary conditions in FEM is easier than FDM. However, FEM has disadvantages such as, requiring enormous processing memory and too much calculations (Matthew & Sadiku, 2001).

Although the governing equations and mechanisms are completely different between radar and seismic waves, most of GPR data processing approaches are derived from seismic data processing. Due to similarities in these two techniques, accordingly we use spectral element method (SEM) in this paper since it has recently gained interest for problems related to 2D (Seriani et al., 1992; Cohen et al., 1993; Priolo et al., 1994) and 3D (Faccioli et al., 1997; Komatitsch and Vilotte, 1998; Seriani, 1998; Komatitsch et al., 2000; Paolucci et al., 1999, Komatitsch et al., 1999c) seismic wave propagation. This technique has been used

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for more than 30 years in computational fluid dynamics (Patera, 1984). This method is almost similar to FEM with a few differences that makes it more optimal than FEM in the time-domain modeling. Indeed, SEM provides an innovative numerical approach to the calculation of synthetic electromagnetic waves in 2D earth models.

This work is the first attempt of applying SEM idea in GPR modeling. In the first part of this paper, the equations of electromagnetic waves are solved based on weak formulation of the problem in SEM framework, then absorbing boundary conditions are implemented in order to attenuate reflections from the edges of the computational domain. The numerical stability and dispersion conditions are also discussed. To verify the efficiency of the SEM in GPR simulation, it is performed on a synthetic inhomogeneous model.

2. Theory

2.1. Spectral element method

The spectral element method (SEM) is a higher-order finite-element method (hp-FEM) that uses the Gauss–Lobatto–Legendre (GLL) nodal points for interpolation functions. The method enables adaptation of the mesh to the irregular surface topography and to the variable wavelengths inside the earth. In SEM, spatial resolution is controlled by two parameters: the typical size of an element, and the polynomial degree, M, used to represent functions on an element, thus each element containing $M + 1$ points in each direction. In this respect, SEMs are related to FEMs based upon a high polynomial (Guo and Babuska, 1986). In practice, if the polynomial degree M is too small, e.g. less than typically 4, a SEM exhibits the same inaccuracies that are observed in a standard FEM applied to wave propagation problems. On the other hand, if the polynomial degree is very large, e.g. greater than 15, the method is spatially very accurate, but the computational requirements become prohibitive. Typically, polynomial degrees between 5 and 10 are optimal for SEM applied to wave propagation problems (Komatitsch and Tromp, 1999). Fig. 1 shows local and global meshes for a four element 2D spectral element discretization with polynomial degree $M = 4$. Also, the spectral element method yields accurate solutions for surface waves without increasing the number of grid points per wavelength, overcoming some of the most severe deficiencies of the finite-difference method (Fichtner, 2011). The formulation of the spectral element method for a structure element is fairly similar to the formulation of the finite-element models (Lee, 2009). However, there are two major differences between SEM and FEM; one of them is choosing interpolation points and the other one is the numerical integration method.

In the standard finite-element method, the equi-spaced nodal points are used, in which increasing number of the nodal points, the interpolation accuracy declines. According to Fig. 2, during wave propagation in 1D homogeneous media, the results show that SEM is very close to analytical solution, which indicates high accuracy of the method. On the other hand, SEM shows much less numerical dispersion compared to FEM because of the high-order polynomial approximations in SEM by moving away from the source. It must be noted that the interpolation function in both methods is Lagrange polynomials. Another difference between FEM and SEM is in the numerical integration quadrature. In the standard FEM, Gaussian quadrature is used while in the SEM, GLL quadrature is used. By using GLL for numerical integration, the mass and damping matrices would be single diagonal due to orthogonality of Lagrange polynomials that leads to high numerical efficiency (Fichtner, 2011).

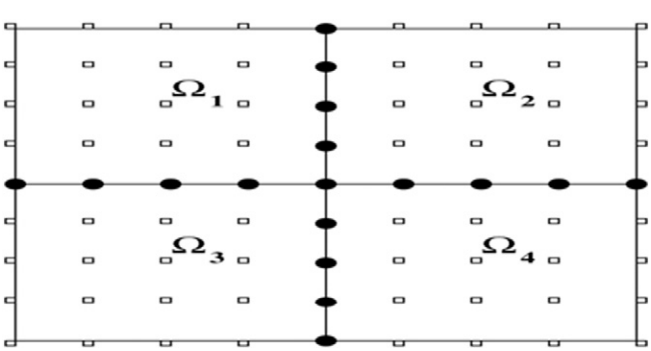

Fig. 1. Discretizing sketch of mesh nodes for a four-element 2D SEM with polynomial degree $M = 4$. Solid circles are shared between elements, and hollow rectangles are for each element. Each spectral element contains $(M + 1)^2 = 25$ Gauss–Lobatto–Legendre points, which from the local mesh for each element. In the global mesh, points lying on edges or corners which are shared between elements.

First, we discretize the computational domain into a number of finite elements, as explained in Fig. 1. Second, we define the nodal degrees-of-freedom (DOFs) and fields at each node of the finite-elements. Then mass, damping and rigidity matrices are made and eventually we do a temporal discretization by central-difference method.

2.2. Governing equations

Maxwell's equations describe the physics of electromagnetic fields, while constitutive relationships quantify material properties. Combining the two provides the foundations for quantitatively describing GPR signals. There are two general approaches to formulating a time-domain finite-element solution. The first approach discretizes Maxwell's equations directly as the first-order partial differential equations. It computes both electric and magnetic fields in the time-domain in a leapfrog fashion and its time-stepping or time-marching process is usually conditionally stable. The second approach discretizes the second-order vector wave equation, or the curl–curl equation, that computes either the electric field or the magnetic field in the time-domain but the process can be made unconditionally stable. The unconditional stability turns out to be very important for many practical applications (Jian-Ming, 2014). Therefore, we will use the second approach to formulate the time-domain finite-element method. Thus by using the second approach the electromagnetic wave equations with a source in a homogeneous medium are given by (Fang et al., 1993; Li, 1994):

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{\nabla^2 \mathbf{E}}{\mu\varepsilon} + \frac{\sigma}{\varepsilon}\frac{\partial \mathbf{E}}{\partial t} = \mathbf{S}_E \tag{1}$$

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} - \frac{\nabla^2 \mathbf{H}}{\mu\varepsilon} + \frac{\sigma}{\varepsilon}\frac{\partial \mathbf{H}}{\partial t} = \mathbf{S}_H \tag{2}$$

E, is the vector electric field (V/m) and **H** is the vector magnetic field (A/m). ε, μ and σ are the dielectric permittivity (F/m), magnetic permeability (H/m) and conductivity (S/m) respectively. When they are scalars, the medium is isotropic and when they are tensors, the medium is anisotropic. The value $\mathbf{S}_E$ is an electric-field source and $\mathbf{S}_H$ is a magnetic-field source and t is time. It is clearly shown that the forms of the differential equations for the magnetic field and the electric field are similar, so to continue our discussion we consider only the electric field under given boundary and initial conditions. The weak form (Jian-Ming, 2014) of Eq. (1) according to the variational principle, is defined as follows,

$$\oint_\Omega \ddot{\mathbf{E}}\,\delta\mathbf{E}\,d\Omega + \oint_\Omega \frac{\sigma}{\varepsilon}\dot{\mathbf{E}}\,\delta\mathbf{E}\,d\Omega + \oint_\Omega \frac{1}{\mu\varepsilon}(\nabla\mathbf{E}.\nabla\delta\mathbf{E})\,d\Omega - \oint_\Gamma \frac{1}{\mu\varepsilon}(\nabla\mathbf{E}.n)\,\delta\mathbf{E}\,d\Gamma = \oint_\Omega \mathbf{S}_E\,\delta\mathbf{E}\,d\Omega \tag{3}$$

where $\ddot{\mathbf{E}}$ and $\dot{\mathbf{E}}$ are the second derivative and the first derivative of the electric field over time, Ω is the region domain, Γ is the region boundary,

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