Journal of Applied Geophysics 116 (2015) 1–9

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

A new approach for density contrast interface inversion based on the parabolic density function in the frequency domain

Lei Shi *, Yonghua Li, Enhui Zhang

Institute of Geophysics, China Earthquake Administration, Beijing 100081, China

ARTICLE INFO

Article history:
Received 23 September 2014
Received in revised form 16 February 2015
Accepted 17 February 2015
Available online 20 February 2015

Keywords:
Gravity data
Parabolic density function
Three-dimensional interface inversion
Frequency domain
Sichuan–Yunnan region

ABSTRACT

Density contrast interface inversion is one of the primary subjects in gravity fields for understanding the Earth's interior structure. In this paper, we presented a new 3D approach for density contrast interface inversion based on the parabolic density function in the frequency domain. The Parabolic density function is adopted to better reflect the real density structure in the subsurface. And the frequency-domain algorithm is utilized to enhance computational efficiency of the forward modeling and inversion. We first derived formula of the frequency-domain parabolic density function, and then presented its procedure for forward modeling and inversion of gravity anomalies to determine density interfaces underground. We also proposed the related techniques for determining the model density parameters and the referencing datum depth, as well as for accelerating convergence. The synthetic data test demonstrated that the new approach and its related techniques are effective and reliable. Finally, we utilized the new approach to obtain the Moho depth distribution in the Sichuan–Yunnan region, China. The result of our approach is consistent well with that from the receiver function, and is better than that from the conventional constant density function approach.

1. Introduction

The density contrast interface inversion of gravity data is an important issue in gravity data inversion, which has great significance for studying basement depth, crustal thickness, and regional tectonics. Bott (1960) first proposed the density interface inversion approach based on the constant density function to obtain the basement depth. Later, Leão et al. (1996) and Barbosa et al. (1997) added depth constraints on the constant density function to reduce the ambiguity of inversion. However, the actual density varies with a number of factors, such as pressure, compaction degree, porosity, rock ages and depth (Athy, 1930). During the process of vertical variation, rock density varies rapidly in the shallows, and then slowly with the depth increasing. Therefore, it is necessary to use a density function varying with depth for the interface inversion, in accordance with the real geology. Cordell (1973) presented an approach for the interface inversion based on an exponential density function to cater to density varying with depth in the real sedimentary basin. Rao (1986) applied a binomial density function in the space domain to 2D gravity modeling, which was later extended to a 3D inversion of gravity data by Gallardo-Delgado et al. (2003). Rao et al. (1993) found that the parabolic density function could not only approximate the real density values, but also give gravity expressions for various geological models. The comparison between the parabolic, binomial and exponential functions shows that the parabolic function better fits most crustal structures (Rao et al., 1994). Chakravarthi and Sundararajan (2004) and Işik and Şenel (2009) applied the density interface inversion approach based on the parabolic density function to obtain good results in agreement with the real geologic situations.

The above approaches for density interface inversion in the space domain modeled the subsurface space as a collection of rectangular prisms, and utilized an iterative algorithm for inversion. However, these approaches involve a large amount of calculation for gravity modeling of these prisms and the repetition of such calculation during the iterative inversion, greatly decreasing the efficiency of inversion. Whereas, the frequency-domain algorithm has the advantage of fast calculation, and thus is widely used in many fields. Parker (1973) first present the frequency-domain approach for forward modeling of gravity interface based on a constant density function, and later Oldenburg (1974) proposed an iterative inversion approach for the density contrast interface based on the constant density function in the frequency domain. Guspi (1992) presented the three-dimensional frequency-domain forward modeling and inversion equations of gravity anomalies based on density contrast polynomials varying with depth, which made the Parker's and Oldenburg's approaches more universal. Ke et al. (2006) employed a frequency-domain approach for density interface inversion based on the exponential density function, and obtained the Moho depth of the Tibetan plateau by using the approach. However, there remains a gap in the frequency-domain inversion relating to the study and application of the density contrast parabolic varying with depth, which can better approximate the actual crustal structure.

* Corresponding author.
E-mail address: shilei@cea-igp.ac.cn (L. Shi).

http://dx.doi.org/10.1016/j.jappgeo.2015.02.022
0926-9851/

In this paper, we presented a new approach for density interface inversion, by combining the merits of the parabolic density function and the frequency domain algorithms presented by Parker (1973) and Oldenburg (1974). Through a series of theoretical formula derivations, we obtained the equation of a frequency-domain parabolic density function, and then we presented its procedure for forward modeling and inversion density interface. We further discussed the related techniques for determining the model density parameters, the referencing datum depth, and the technique for accelerating convergence, as well as their effects on the inversion results. We tested the effectiveness of the new approach and its related techniques on the synthetic data. Finally, we performed the new approach on the regional gravity anomalies in the Sichuan–Yunnan region to obtain the Moho depth distribution, and then compared the result with those from the receiver function and the conventional constant density function approach.

2. Method

2.1. Frequency-domain forward modeling of the interface based on the parabolic density function

Suppose the density contrast of the target interface varies with depth and can be approximated well by a parabolic function as (Rao et al., 1993):

$$\sigma(\zeta)=\frac{{\sigma_0}^3}{(\sigma_0-\alpha\zeta)^2} \tag{1}$$

where $\sigma(\zeta)$ is the density function at depth ζ, σ_0 is the density contrast on the ground, and α is the density attenuation coefficient. In the practical application, σ_0 and α usually can be determined according to the known values of density at some different depths.

Then, by using a right-hand coordinate system, the gravity anomaly at an arbitrary station $(x,y,0)$ on the regular observational geometry caused by the residual mass above the target interface can be expressed as:

$$\Delta g(x,y,0)=G\iiint_V\frac{\sigma(\xi,\eta,\zeta)\zeta}{\left[(\xi-x)^2+(\eta-y)^2+\zeta^2\right]^{\frac{3}{2}}}d\xi d\eta d\zeta. \tag{2}$$

where G is the universal gravitation constant, V is the whole volume of the residual mass and $d\xi d\eta d\zeta$ is an element volume for integral.

Substituting the parabolic density function (Eq. (1)) into Eq. (2) yields:

$$\Delta g(x,y,0)=G\iiint_V\frac{{\sigma_0}^3(\xi,\eta)\zeta}{(\sigma_0(\xi,\eta)-\alpha\zeta)^2\left[(\xi-x)^2+(\eta-y)^2+\zeta^2\right]^{\frac{3}{2}}}d\xi d\eta d\zeta. \tag{3}$$

Applying the Fourier transform to Eq. (3), we obtain:

$$F[\Delta g]=G\int\int_{-\infty}^{+\infty}\frac{\zeta}{\left[(\xi-x)^2+(\eta-y)^2+\zeta^2\right]^{\frac{3}{2}}}e^{-i(ux+vy)}dxdy\iiint_V\frac{{\sigma_0}^3(\xi,\eta)}{(\sigma_0(\xi,\eta)-\alpha\zeta)^2}d\xi d\eta d\zeta. \tag{4}$$

where u and v are the wave numbers along x and y directions, respectively.

According to the convolution theorem and the Hankel transform, there is:

$$F[\Delta g]=2\pi Ge^{-kz_0}\iint_D{\sigma_0}^3(\xi,\eta)e^{-i(u\xi+v\eta)}d\xi d\eta\int_0^h\frac{e^{-\sqrt{u^2+v^2}\zeta}}{(\sigma_0(\xi,\eta)-\alpha(\zeta+z_0))^2}d\zeta. \tag{5}$$

If we let $\sqrt{u^2+v^2}=k$, and apply the Taylor expansion $e^{-\sqrt{u^2+v^2}\zeta}=e^{-k\zeta}$ at $\zeta=0$, we have:

$$F[\Delta g]=2\pi Ge^{-kz_0}\iint_D{\sigma_0}^3(\xi,\eta)e^{-i(u\xi+v\eta)}d\xi d\eta\sum_{n=0}^{\infty}\frac{(-k)^n}{n!}\int_0^h\frac{\zeta^n}{(\sigma_0(\xi,\eta)-\alpha(\zeta+z_0))^2}d\zeta. \tag{6}$$

Integrating Eq. (6) over ζ, we finally obtain the frequency spectrum of the gravity anomaly:

$$F[\Delta g]=\frac{2\pi Ge^{-kz_0}}{\alpha^2}\left\{F[L1]+kF[L2]+\sum_{n=2}^{\infty}\frac{(-k)^n}{n!}\left\{F[L3]+nF[L4]+nF[L5]+n\sum_{i=2}^{n-1}\frac{F[L6]}{i}\right\}\right\}. \tag{7}$$

where $L1=\frac{{\sigma_0}^3(\xi,\eta)\Delta h}{b(b-\Delta h)}$, $L2={\sigma_0}^3(\xi,\eta)\left(\frac{\Delta h}{\Delta h-b}-\ln\left(1-\frac{\Delta h}{b}\right)\right)$, $L3=-\frac{{\sigma_0}^3(\xi,\eta)\Delta h^n}{\Delta h-b}$, $L4={\sigma_0}^3(\xi,\eta)\Delta h$, $L5={\sigma_0}^3(\xi,\eta)\ln\left(1-\frac{\Delta h}{b}\right)$, $L6={\sigma_0}^3(\xi,\eta)\Delta h^i$, $b=\frac{\sigma_0(\xi,\eta)}{\alpha}-z_0$, and Δh is the relief of the interface.

Then we can obtain the gravity anomaly in the space domain through performing a reverse Fourier transform on Eq. (7).

2.2. Frequency-domain inversion of the interface based on the parabolic density function

The basic workflow of our inversion is stated below. We first establish an initial model according to the observed gravity anomaly. Then we conduct forward modeling in the frequency domain based on the parabolic density function, and calculate the least-square deviation between the theoretical gravity anomaly and the observed one. And then according to the deviation, we modify the initial model, and redo-forward modeling and comparisons until the deviation smaller than the tolerance. The detailed procedure for our inversion approach is listed as follows:

1) Read in the observed anomalies Δg, the initial density parameters in Eq. (1) and the referencing datum depth, and transform the observed gravity data into the frequency domain to get $F[\Delta g]$.
2) Use Eq. (8) to calculate the initial depth of the target interface $\Delta h^{(0)}$ where $p=F^{-1}\left[\alpha^2e^{kz_0}F[\Delta g]/2\pi G\times f(k)\right]$, and $f(k)$ is a filter factor.

$$\Delta h^{(0)}=\frac{pb^2\big/{\sigma_0}^3}{1+pb\big/{\sigma_0}^3} \tag{8}$$

3) Perform forward modeling using the initial depth $\Delta h^{(0)}$ and Eq. (7), yielding the theoretical gravity anomaly in the frequency domain $F[\Delta g]^{(1)}$, and then obtaining the space-domain one $\Delta g^{(1)}$ via the inverse Fourier transform.
4) Use Eq. (9) below to modify the initial depth $\Delta h^{(0)}$, obtaining the modification Δh, where $q=F^{-1}\left[\left(F[\Delta g]-F\left[\Delta g^{(1)}\right]\right)\alpha^2e^{kz_0}/2\pi G\times f(k)\right]$.

$$\Delta h=\frac{qb^2\big/{\sigma_0}^3}{1+qb\big/{\sigma_0}^3} \tag{9}$$

5) Calculate the new depth of the target interface $\Delta h^{(1)}$ from $\Delta h^{(1)}=\Delta h^{(0)}+\Delta h\times\lambda$, where λ is an accelerating convergence factor.
6) Calculate the least square deviation between the theoretical gravity anomaly and the observe one, i.e., $||\Delta g-\Delta g^{(1)}||_2$.
7) Judge whether the deviation is smaller than the tolerance. If not, repeat steps 3) to 6) until it is attained.
8) When the convergence is reached after n times of iteration, the final interface depth is $h=\Delta h^{(n)}+z_0$.