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Theoretical relationship between elastic wave velocity and electrical resistivity

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ABSTRACT

Elastic wave velocity and electrical resistivity have been commonly applied to estimate stratum structures and obtain subsurface soil design parameters. Both elastic wave velocity and electrical resistivity are related to the void ratio; the objective of this study is therefore to suggest a theoretical relationship between the two physical parameters. Gassmann theory and Archie's equation are applied to propose a new theoretical equation, which relates the compressional wave velocity to shear wave velocity and electrical resistivity. The piezo disk element (PDE) and bender element (BE) are used to measure the compressional and shear wave velocities, respectively. In addition, the electrical resistivity is obtained by using the electrical resistivity probe (ERP). The elastic wave velocity and electrical resistivity are recorded in several types of soils including sand, silty sand, silty clay, silt, and clay–sand mixture. The appropriate input parameters are determined based on the error norm in order to increase the reliability of the proposed relationship. The predicted compressional wave velocities from the shear wave velocity and electrical resistivity are similar to the measured compressional velocities. This study demonstrates that the new theoretical relationship may be effectively used to predict the unknown geophysical property from the measured values.

1. Introduction

The geophysical method, which includes an elastic wave survey, electrical resistivity method, and gravity survey, is advantageous for investigating the structures of a subsurface (Reynolds, 2003). In particular, the elastic wave and electrical resistivity methods are known to be efficient tools due to their simple procedures and high resolution. The two techniques are applied to investigate the natural conditions of soil, contaminated media, and faults in rock. However, site conditions often render the application of these methods difficult for exploration purposes. The two techniques may be used simultaneously as complementary methods to resolve the difficulty of a single exploration. Popp and Kern (1993) analyzed thermal conductivity in a subsurface by using both the elastic wave and electrical resistivity methods. Ichiki et al. (2006) also used the two techniques to deduce the water content and profiles of geothermal heat.

Shankar and Riedel (2011) verified the reliability of measured elastic wave velocity through electrical resistivity in gas-hydrated deposits. However, the verification was performed by using the relative trends of increase and decrease in the data because the two data sets reflect different physical characteristics. In order to perform a reasonable comparison, Faust (1953) suggested a relationship between the compressional wave velocity and the formation factor, which is the ratio of the electrical resistivity of the soil particles to that of the pore water. Hacikoylu et al. (2006) proposed a similar equation, including compressional wave velocity and formation factor, suitable for unconsolidated sediments, because the technique suggested by Faust (1953) is limited to use in consolidated sandstone. Hubert (2008) presented an equation that takes into account the ionic characterization for clay, based on the method by Hacikoylu et al. (2006). Moreover, Han et al. (2011) provided a linear relationship between the compressional wave velocity and the formation factor by using the measured data of 63 specimens of sandstone; their equation exhibits a high coefficient of determination. However, it is difficult to apply the above-mentioned equations to a variety of soils, because they are based on empirical relationships and the shear wave velocity is omitted. The shear wave velocity is used because it provides a better estimate of the particle fabric of soil than is available from the compressional wave velocity (Santamarina et al., 2001). Therefore, for superior reliability, an equation should incorporate the compressional wave, shear wave, and electrical resistivity, together with the theoretical background.

In this study, a theoretically correlated equation between the elastic wave velocity and electrical resistivity is proposed. The theoretical concepts regarding elastic wave velocity and electrical resistivity were addressed, and common parameters between velocity and resistivity

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were also explained to propose the new relationship. The measurement system for the elastic wave velocity and electrical resistivity were described separately and the reliability of the suggested equation was analyzed. In addition, other equations were compared with the new proposed equations. Finally, a summary and conclusions are described.

2. Elastic wave velocity and electrical resistivity

The elastic wave velocities and electrical resistivity are related to the various physical properties of soil, and a mathematical description of this relationship is provided below.

2.1. Electrical resistivity

The electrical resistivity is a parameter that opposes the flow of current in a material, and it is the reciprocal of electrical conductivity. Therefore, electrical conductivity is generally mentioned in order to understand the characterization of electrical resistivity in a medium. The electrical conductivity (σ_{mix}) in a soil mixture consists of three components: the conductivities of the soil particle (σ_P), the electrolyte (σ_{el}), and the specific surface (S_a), as shown in Eq. (1) (Santamarina et al., 2001).

$$\sigma_{mix} = (1-n) \cdot \sigma_P + n \cdot \sigma_{el} + (1-n) \cdot \frac{\gamma_P}{g} \cdot \lambda_{ddl} \cdot S_a \tag{1}$$

where n denotes porosity, γ_P is unit weight, g is gravity, and λ_{ddl} is the surface conduction.

The specific surface conductivity decreases to zero when a medium is composed of large particles. In the case of a coastal area, for which the conductivity of electrolyte is high, the soil particle conductivity can be ignored owing to the small value. Finally, Eq. (1) is rearranged to Eq. (2) and converted into an electrical resistivity equation (E_{mix} and E_{el}), as shown in Eq. (3). Eq. (3) is called Archie's (1942) law, and the two factors incorporating the effects of cementation (α) and shape (m) are empirical constants.

$$\sigma_{mix} = n \cdot \sigma_{el} \tag{2}$$

$$E_{mix} = \alpha \cdot E_{el} \cdot n^{-m} \tag{3}$$

where the cementation factor (α) depends on the tortuosity in a porous material, and takes values in the range of 0.6–3.5 (Salem and Chilingarian, 1999). The shape factor (m) depends on the shape of the particle, the porous structure, and the specific surface, and generally has a value in the range of 1.4–2.2 (Abu-Hassanein et al., 1996; Salem and Chilingarian, 1999).

2.2. Elastic wave velocity

Theories regarding elastic wave propagation in a saturated medium have been suggested by Wood (1949), Gassmann (1951) (translated into English by Berryman, 1999), and Biot (1956). These theories have been applied to estimate the properties of soil, including the void ratio (Foti et al., 2002) and crack density (Tang, 2011). Lee and Yoon (2014) analyzed the reliability of the suggested equation by using a statistical method, and the results show that the Gassmann theory provides a high resolution. Therefore, the Gassmann theory was applied to investigate the relationship between elastic wave velocity and electrical resistivity with reference to the previous study. The elastic wave propagation was defined with the bulk modulus in an isotropic porous medium because the bulk modulus effectively reflects the volume change generated by wave propagation. The compression and expansion of the fluid owing to wave propagation are also considered to predict the change in behavior with fluid pressure. The chemical interaction that occurs between a porous system and the fluid is neglected. The suggested equation is written as follows.

$$B_{Gassmann} = B_{sk} + \frac{\left(1-\frac{B_{sk}}{B_g}\right)}{\left(\frac{n}{B_f} - \frac{B_{sk}}{B_g^2} + \frac{1-n}{B_g}\right)} \tag{4}$$

where $B_{Gassmann}$ means the bulk modulus of the mixture. B_{sk}, B_g, and B_f are the bulk moduli of the soil skeleton, soil grain, and fluid, respectively. n denotes the porosity of the medium.

The bulk modulus of a mixture can be expressed in terms of mass density ρ, compressional wave velocity V_P, and shear wave velocity V_S, as shown by Eq. (5). Therefore, the equation suggested by Gassmann (1951) is a function of the elastic wave velocity, as shown by Eq. (6).

$$B_{Gassmann} = \rho \cdot \left(V_P^2 - \frac{4}{3} \cdot V_S^2\right) \tag{5}$$

$$B_{sk} + \frac{\left(1-\frac{B_{sk}}{B_g}\right)}{\left(\frac{n}{B_f} - \frac{B_{sk}}{B_g^2} + \frac{1-n}{B_g}\right)} = \rho \times \left(V_P^2 - \frac{4}{3} \times V_S^2\right) \tag{6}$$