# High-order nonlinear solver for multiple roots 

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#### Abstract

A method of order four for finding multiple zeros of nonlinear functions is developed. The method is based on Jarratt's fifth-order method (for simple roots) and it requires one evaluation of the function and three evaluations of the derivative. The informational efficiency of the method is the same as previously developed schemes of lower order. For the special case of double root, we found a family of fourth-order methods requiring one less derivative. Thus this family is more efficient than all others. All these methods require the knowledge of the multiplicity. Published by Elsevier Ltd


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## 1. Introduction

There is a vast literature on the solution of nonlinear equations and nonlinear systems, see for example Ostrowski [1], Traub [2], Neta [3] and references there. Here we develop a high-order fixed point type method to approximate a multiple root. There are several methods for computing a zero $\xi$ of multiplicity $m$ of a nonlinear equation $f(x)=0$, see Neta [3]. Newton's method is only of first order unless it is modified to gain the second order of convergence, see Rall [4] or Schröder [5]. This modification requires a knowledge of the multiplicity. Traub [2] has suggested the use of any method for $f^{(m)}(x)$ or $g(x)=\frac{f(x)}{f^{\prime}(x)}$. Any such method will require higher derivatives than the corresponding one for simple zeros. Also the first one of those methods requires the knowledge of the multiplicity $m$. In such a case, there are several other methods developed by Hansen and Patrick [6], Victory and Neta [7], and Dong [8]. Since in general one does not know the multiplicity, Traub [2] suggested a way to approximate it during the iteration.

For example, the quadratically convergent modified Newton's method is

$$
\begin{equation*}
x_{n+1}=x_{n}-m \frac{f_{n}}{f_{n}^{\prime}} \tag{1}
\end{equation*}
$$

and the cubically convergent Halley's method [9] is

$$
\begin{equation*}
x_{n+1}=x_{n}-\frac{f_{n}}{\frac{m+1}{2 m} f_{n}^{\prime}-\frac{f_{n} f_{n}^{\prime \prime}}{2 f_{n}^{\prime \prime}}} \tag{2}
\end{equation*}
$$

[^0]where $f_{n}^{(i)}$ is short for $f^{(i)}\left(x_{n}\right)$. Another third-order method was developed by Victory and Neta [7] and is based on King's fifth-order method (for simple roots) [10]
\[

$$
\begin{align*}
& w_{n}=x_{n}-\frac{f_{n}}{f_{n}^{\prime}} \\
& x_{n+1}=w_{n}-\frac{f\left(w_{n}\right)}{f_{n}^{\prime}} \frac{f_{n}+A f\left(w_{n}\right)}{f_{n}+B f\left(w_{n}\right)} \tag{3}
\end{align*}
$$
\]

where

$$
\begin{align*}
& A=\mu^{2 m}-\mu^{m+1} \\
& B=-\frac{\mu^{m}(m-2)(m-1)+1}{(m-1)^{2}} \tag{4}
\end{align*}
$$

and

$$
\begin{equation*}
\mu=\frac{m}{m-1} \tag{5}
\end{equation*}
$$

Yet two other third-order methods developed by Dong [8], both require the same information and both are based on a family of fourth-order methods (for simple roots) due to Jarratt [11]:

$$
\begin{align*}
& x_{n+1}=x_{n}-u_{n}-\frac{f\left(x_{n}\right)}{\left(\frac{m}{m-1}\right)^{m+1} f^{\prime}\left(x_{n}-u_{n}\right)+\frac{m-m^{2}-1}{(m-1)^{2}} f^{\prime}\left(x_{n}\right)}  \tag{6}\\
& x_{n+1}=x_{n}-\frac{m}{m+1} u_{n}-\frac{\frac{m}{m+1} f\left(x_{n}\right)}{\left(1+\frac{1}{m}\right)^{m} f^{\prime}\left(x_{n}-\frac{m}{m+1} u_{n}\right)-f^{\prime}\left(x_{n}\right)} \tag{7}
\end{align*}
$$

where $u_{n}=\frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}$.
Our starting point here is Jarratt's method [12] given by the iteration

$$
\begin{equation*}
x_{n+1}=x_{n}-\frac{f\left(x_{n}\right)}{a_{1} f^{\prime}\left(x_{n}\right)+a_{2} f^{\prime}\left(y_{n}\right)+a_{3} f^{\prime}\left(\eta_{n}\right)} \tag{8}
\end{equation*}
$$

where $u_{n}$ is as above and

$$
\begin{align*}
y_{n} & =x_{n}-a u_{n} \\
v_{n} & =\frac{f\left(x_{n}\right)}{f^{\prime}\left(y_{n}\right)}  \tag{9}\\
\eta_{n} & =x_{n}-b u_{n}-c v_{n} .
\end{align*}
$$

Jarratt has shown that this method (for simple roots) is of order 5 [12] if the parameters are chosen as follows

$$
\begin{equation*}
a=1, \quad b=\frac{1}{8}, \quad c=\frac{3}{8}, \quad a_{1}=a_{2}=\frac{1}{6}, \quad a_{3}=\frac{2}{3} . \tag{10}
\end{equation*}
$$

It requires one function- and three derivative-evaluation per step. Thus the informational efficiency (see [2]) is 1.25 . Since Jarratt did not give the asymptotic error constant, we have employed Maple [13] to derive it,

$$
\frac{1}{24} A_{5}+\frac{1}{2} A_{4} A_{2}-\frac{1}{4} A_{3}^{2}+\frac{1}{8} A_{2}^{2} A_{3}+A_{2}^{4}
$$

where $A_{i}$ are given by (14) with $m=1$.

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