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The air-cargo consolidation problem with pivot weight: Models and solution methods

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ABSTRACT

In the international air cargo business, freight is usually consolidated into containers, called Unit Load Devices (ULDs). The transportation charge of a ULD depends on whether the total weight exceeds a certain threshold, called the *pivot weight*. If the total load tendered by a freight forwarder is less than the pivot weight, it gets charged at the *under-pivot rate*. Any portion of the load that exceeds the pivot weight is charged at the *over-pivot* rate. This scheme is adopted for safety reasons to avoid the overloading of ULDs. We formulate a mixed integer program, and propose four solution methodologies for the air-cargo consolidation problem under the pivot-weight scheme (ACPW). These are exact solution approaches based on branch-and-price, a best-fit decreasing loading heuristic, and two extended local branching heuristics (a multi-local tree search and a relaxation-induced neighborhood search). The local branching heuristic with relaxation-induced neighborhood search is found to outperform other approaches in terms of solution quality and computational time. Problems with up to 400 shipments and 80 containers are solved to within 3.4% of optimality in less than 20 min.

1. Introduction

The operations of an airfreight forwarder include making capacity reservations with cargo airlines, consolidating shipments, tendering freight to the airline, and breaking bulk for the delivery to the final destination. Instead of reserving in terms of chargeable weight, large forwarders sometimes reserve a number of containers from airlines. These containers are called Unit Load Devices (ULDs). Airlines offer several types of containers with different capacity and cost characteristics.

The air-cargo consolidation problem under the pivot-weight scheme (ACPW) is characterized by a *pivot weight* and two unit costs, respectively, the *under-pivot rate* and the *over-pivot rate*. The pivot weight U_j is a weight threshold of a ULD, below which the cargo is charged the under-pivot rate. Any weight that falls between the pivot weight and the maximum capacity of the ULD is charged at a special unit rate, *higher* than the pivot rate, called the over-pivot rate. In addition, there is a fixed reservation cost for each ULD. Faced with this pricing scheme, a forwarder is interested in finding the optimal consolidation decisions to minimize total cost. This problem is commonly encountered by large freight forwarders.

Note that the pivot-weight scheme discussed here is not a discount. Airlines price in this way to prevent shippers from overloading ULDs. The over-pivot cost can be seen as an incremental *penalty*, rather than a discount. The well-known "bumping-clause", as in Bookbinder and Higginson [2], whereby it can be advantageous to over-declare the total weight dispatched, is used more for general cargo (goods that are not containerized). Instead, the pivot-weight scheme is suitable for shipments consolidated in ULDs.

An airfreight forwarder's problem is thus a shipment consolidation application where decisions on consolidation and timing are made: which loads or shipments will be combined for eventual transport together? and when will that consolidated load be dispatched? Instead of sending palletized shipments by truck, an airfreight forwarder sends goods in a ULD on a plane. And rather than benefiting from a discount for larger loads, the forwarder will, as noted, pay a penalty when the weight exceeds U_j. The ACPW problem thus models a practical situation that, other than Li et al. [10], has previously received limited study. Li et al. [10] give a detailed description of the problem and propose a large-scale neighborhood search heuristic to solve it. In this paper, we provide two formulations. One is very similar to the model of Li et al. [10] but eliminates a redundant constraint, and the other is based on column generation. We then propose four solution methodologies to solve the problem that we test and compare.

2. Literature review

Earlier work on the airfreight consolidation problem assumes that forwarders reserve cargo space from airlines in terms of

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payload or chargeable weight. This assumption is applicable to general cargo and non-palletized shipments. However, large airfreight forwarders usually make their booking in terms of ULDs.

Other than Li et al. [10], the pivot-weight scheme has not been investigated before. There is, however, some research on the ocean counterpart that addresses the loading problem based on 20-foot containers. Pisinger [13] examines an ocean container-loading problem, where a subset of rectangular boxes have to be loaded into a rectangular container of fixed dimensions, such that the volume of the packaged boxes is maximized. Brønmo et al. [3] present a Dantzig–Wolfe procedure for the ship-scheduling problem with flexible cargo sizes. Although work concerning ocean cargo may give some insight to the problem under study, ocean containers have a conventional fixed capacity limit, instead of a pivot weight.

The ACPW application also differs from the bin-packing and knapsack problems which have been used as the foundation of many consolidation-related problem variants. In bin-packing, cost is associated with the bin-level, but no charge is laid based on the weight or volume of items as in ACPW. The decision variables for bin-packing are all binary. Our model, however, also has a *continuous* "overage" weight or capacity compared to the variable-size-and-cost bin-packing problem. In addition, we have distinct capacities on different bins. The open-end bin-packing problem [9] allows the capacity of each bin to be exceeded by only one item; the ACPW does not have such a restriction. Our case also differs from the conventional knapsack problem: first, we have multiple knapsacks with diverse capacities; second, we have an over-pivot rate per unit of cargo that exceeds the pivot weight.

There is some existing research on bin-packing problems which gives much insight to tackle our application. In particular, the variable-size bin-packing problem (VSBPP) resembles our work, but without the continuous variables that represent the over-pivot weight. However, it should be noted that an important assumption was made in that literature with respect to the fixed cost per bin: bins of larger capacity have a proportionally greater fixed cost. Although this assumption yields an easy approximation, it does not reflect reality in the transportation industry. In the airfreight business, the ULD reservation fee is a relatively independent attribute, one that may or may not be correlated to its capacity. Therefore we lift this hypothesis of the VSBPP in our ACPW, by considering a fixed reservation cost independent of ULD capacity. Crainic et al. [4] recognize the independence of the fixed reservation cost and the bin capacity, but do not account for the unit cost c_j (i.e. the under-pivot rate) or the over-pivot rate c_j^E.

The remainder of this paper is organized as follows: Section 3 presents the mathematical model for ACPW. Sections 4 and 5 introduce a branch-and-price algorithm, as well as a best-fit decreasing heuristic to the ACPW problem. This is followed by two extensions of local branching applied to the ACPW problem in Section 6. The computational performance of the four solution approaches is compared in Section 7. Section 8 summarizes our conclusions, and offers suggestions for further research.

3. Problem formulation

In the airfreight consolidation problem with pivot weight, a freight forwarder is faced with the decision to allocate a total of m shipments. Each shipment $i \in I$ (I is the set of shipments), with gross weight g_i, is to be allocated to a particular ULD $j \in J$ (J is the set of reserved ULDs), subject to a capacity limitation. Suppose there are n ULDs, each with a fixed reservation cost f_j, a pivot capacity U_j, an extra-pivot capacity U_j^E, an under-pivot rate c_j and an over-pivot rate c_j^E. The ULD thus has a total weight capacity of $U_j + U_j^E$. Note that the reservation cost f_j is dependent on the time the ULD is reserved, being low at first when few ULDs are served and getting higher as the number of ULDs reserved increases. As in Li et al. [10], we do not model time explicitly in this work and we assume that it is constant.

We use binary decision variables x_{ij} and z_j, where x_{ij} takes value 1 if shipment i is assigned to ULD j, and 0 otherwise; z_j takes value 1 if ULD j is used, and 0 otherwise. Our formulation employs continuous variables y_j^E to denote the additional capacity beyond the pivot weight for ULD j. The air-cargo consolidation problem with pivot weight is thus modeled as

$$[\text{ACPW}]: \min \quad \sum_j f_j z_j + \sum_i \sum_j g_i c_j x_{ij} + \sum_j c_j^E y_j^E \tag{1}$$

$$\text{s.t.} \quad \sum_j x_{ij} = 1 \quad \forall i \in I \tag{2}$$

$$\sum_i g_i x_{ij} \le U_j z_j + y_j^E \quad \forall j \in J \tag{3}$$

$$y_j^E \le U_j^E z_j \quad \forall j \in J$$
$$x_{ij} \in \{0,1\}, \quad z_j \in \{0,1\}, \quad y_j^E \ge 0 \quad \forall i,j \tag{4}$$

The objective (1) minimizes the fixed reservation cost plus the under-pivot and over-pivot costs. Constraints (2) require that each shipment be assigned to exactly one ULD. Relations (3) and (4) model the pivot capacity and over-pivot capacity for each ULD j. Li et al. [10] showed that this problem is NP-hard, as it can be reduced to the well-known 3-partition problem.

We note that in the formulation of Li et al. [10], the constraints $\sum_i x_{ij} \le N z_j$ $\forall j$ are redundant.

4. Branch-and-price

By relaxing constraint (2), we obtain the following subproblem:

$$[\text{ACPW} - \text{Relax}]: \min \quad \sum_j f_j z_j + \sum_i \sum_j g_i c_j x_{ij}$$
$$+ \sum_j c_j^E y_j^E + \sum_i \lambda_i \left(1 - \sum_j x_{ij}\right) \tag{5}$$

$$\text{s.t.} \quad \sum_i g_i x_{ij} \le U_j z_j + y_j^E \quad \forall j \tag{6}$$

$$y_j^E \le U_j^E z_j \quad \forall j$$
$$x_{ij} \in \{0,1\}, z_j \in \{0,1\}, \quad y_j^E \ge 0 \quad \forall i,j \tag{7}$$

Note that since we relaxed an equality constraint, the multiplier λ_i is unrestricted in sign. The preceding subproblem can be decomposed to n subproblems as follows:

$$[SP_j]: W_j = \min \quad f_j z_j + \sum_i (g_i c_j - \lambda_i) x_{ij} + c_j^E y_j^E$$
$$\text{s.t.} \quad \sum_i g_i x_{ij} \le U_j z_j + y_j^E \tag{8}$$

$$y_j^E \le U_j^E z_j$$
$$x_{ij} \in \{0,1\} \quad \forall i, \quad z_j \in \{0,1\}, \quad y_j^E \ge 0 \tag{9}$$

$[SP_j]$ is a 0/1 knapsack problem with one continuous variable y_j^E and an additional constraint (9), corresponding to each ULD j. The Lagrangian bound is given by $\sum_j W_j + \sum_i \lambda_i$. The best Lagrangian bound is thus $\max_\lambda \sum_j W_j + \sum_i \lambda_i$, which is equivalent to the Lagrangian master problem:

$$[\text{MP}]: \max \quad \sum_i \lambda_i + \sum_j \theta_j$$
$$\text{s.t.} \quad \theta_j + \sum_i \lambda_i x_{ij}^h \le f_j z_j^h + \sum_i g_i c_j x_{ij}^h + c_j^E y_j^{Eh} \quad \forall h,j \tag{10}$$