Computers & Operations Research 56 (2015) 110–119

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Incorporating location, inventory and price decisions into a supply chain distribution network design problem

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ARTICLE INFO

Available online 15 September 2014

Keywords:
Supply chain management
Distribution network design
Integration
Location-inventory problem
Price-sensitive demands
Lagrangian relaxation

ABSTRACT

The goal of this paper is to study a profit-maximization location-inventory problem in a multi-commodity supply chain distribution network with price-sensitive demands. The problem determines location, allocation, price and order-size decisions in order to maximize the total profit of serving the customers. The problem is formulated as a mixed-integer nonlinear programming model and solved using a Lagrangian relaxation algorithm for the two cases of uncapacitated and capacitated distribution centers. The computational results show that the quasi-optimality tolerance is reasonable and the computational time is very small for solving large-sized instances of the problem.

1. Introduction

For at least thirty years, and probably as far back as the mid-seventies, considerable attention has been paid to *profit-maximization location-allocation problems* (PM-LAPs). PM-LAPs can be classified into those with price-sensitive demands and those with demand flexibility, where the price only affects customers' decisions on whether or not to get a service, and where the demand volumes are independent of the price decisions. Wagner and Falkson [1] proposed a PM-LAP of the first category, but they did not offer a solution method. Hansen et al. [2] and Hanjoul et al. [3] studied similar PM-LAPs and presented methods for solving them. Hansen et al. [4] considered a more complicated problem under zone pricing. Ahmadi-Javid and Ghandali [5] presented a capacitated PM-LAP with price-sensitive demands. PM-LAPs with demand flexibility have also been studied in certain papers, e.g. [6,7].

Although PM-LAPs have been well examined, the literature on integrated location problems with profit-maximization objectives is hard to find. *Location-inventory problems* (LIPs) are integrated location problems that determine location and inventory decisions simultaneously. LIPs have received much attention in the literature over the past decade; however, most investigations have cost-minimization objectives; see, e.g. [8–21]. Shen [22] and Shu et al. [23] considered *profit-maximization location-inventory problems* (PM-LIPs) with demand flexibility, but to date no PM-LIP with price-sensitive demands has been considered. This led us to study a PM-LIP with price-sensitive demands in a multi-commodity supply chain. The problem is studied for two cases: uncapacitated and capacitated distribution centers. The problem is formulated as a mixed-integer nonlinear program and solved by a Lagrangian relaxation algorithm for each case.

The rest of the paper is organized as follows. Section 2 states and formulates the uncapacitated PM-LIP, and Section 3 presents a Lagrangian relaxation algorithm for solving this problem. Section 4 considers the capacitated PM-LIP. Section 5 reports the computational experience, and Section 6 concludes the paper.

2. Uncapacitated PM-LIP

The PM-LIP under consideration is first described in Section 2.1, and is then formulated in Section 2.2.

2.1. Problem statement

Consider an LIP that simultaneously determines the location of *distribution centers* (DCs), the allocation of customers to open DCs, order-size decisions at open DCs, and retail-price decisions of commodities offered at each DC, in order to maximize the profit of a supply chain distribution network with price-sensitive customer demands. Given that the wholesale price (cost price) is known, deciding the retail price (selling price) at each DC is equivalent to determining its corresponding markup, which is the ratio of the profit per unit (the difference between the retail price and the wholesale price) to the wholesale price. Hence, the markup, instead of the retail price, is considered as the unknown value that must be determined for any commodity offered at

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http://dx.doi.org/10.1016/j.cor.2014.07.014
0305-0548/

each DC. The underlying assumptions of the problem are as follows:

(1) Each DC distributes different commodities that may vary with DC.
(2) At each DC, commodities are independently ordered from the suppliers.
(3) No fractional assignment is allowed; that is, each customer's demand for each commodity must be supplied only from one open DC.
(4) Each DC, for every commodity, has a finite number of possible scenarios for the markup, which are called markup levels. Each DC offers a unique retail price to all of its allocated customers for each commodity. Moreover, wholesale prices are given and may vary with DC.
(5) Each customer's demand for each commodity depends on the retail price offered by the DC which is allocated to it.
(6) It is not mandatory to supply all customers' demands; an arbitary subset of customers can be selected and served for each commodity.
(7) The inventory system at each open DC follows the continuous-review inventory policy, which is also known as the (Q,R) inventory policy, where for each commodity the fixed quantity Q is ordered from the supplier as the inventory on hand at the DC falls to or below the reorder point R.
(8) For each commodity, a fixed cost for placing an order and a holding cost for working inventory must be paid at each open DC.

2.2. Problem formulation

The sets, parameters and decision variables used in the formulation are given in the first three subsections and the formulation is given in the last subsection.

2.2.1. Sets

- I the set of potential DCs
- J the set of customers
- G the set of markup levels
- K the set of commodities

2.2.2. Parameters

- f_i the yearly fixed cost for establishing DC i for $i \in I$
- t_{ijk} the transportation cost per unit of commodity k between DC i and customer j for $i \in I,\ j \in J,\ k \in K$
- o_{ik} the fixed cost of placing an order for commodity k at DC i for $i \in I,\ k \in K$
- e_{ik} the fixed transportation cost between the supplier of commodity k and DC i for $i \in I,\ k \in K$
- a_{ik} the transportation cost per unit between the supplier of commodity k and DC i for $i \in I,\ k \in K$
- h_{ik} the yearly holding cost per unit of commodity k at DC i for $i \in I,\ k \in K$
- c_{ik} the procurement cost (wholesale price) per unit of commodity k at DC i for $i \in I,\ k \in K$
- b_{igk} the gth markup of commodity k considered by DC i for $i \in I,\ g \in G,\ k \in K$
- p_{igk} the retail price per unit of commodity k at DC i with markup level g, which equals $(1+b_{igk})c_{ik}$ for $i \in I,\ g \in G,\ k \in K$
- d_{igjk} the yearly demand of customer j for commodity k from DC i when the gth markup level is offered, for $i \in I,\ g \in G,\ j \in J,\ k \in K$

2.2.3. Decision variables

- X_i a binary variable that takes 1 if DC i is established, and 0, otherwise, for $i \in I$
- Z_{igk} a binary variable that takes 1 if commodity k is offered by DC i with markup level g, and 0, otherwise, for $i \in I,\ g \in G,\ k \in K$
- Y_{igjk} a binary variable that takes 1 if customer j is assigned to DC i with markup level g to supply its demand for commodity k, for $i \in I,\ g \in G,\ j \in J,\ k \in K$
- Q_{igk} a non-negative real-valued variable representing the order size of commodity k at DC i with markup level g for $i \in I,\ g \in G,\ k \in K$

2.2.4. Formulation

The formulation of the uncapacitated PM-LIP is as follows:

$$\max \sum_{i\in I}\sum_{g\in G}\sum_{j\in J}\sum_{k\in K} p_{igk}d_{igjk}Y_{igjk} - \sum_{i\in I} f_iX_i - \sum_{i\in I}\sum_{g\in G}\sum_{j\in J}\sum_{k\in K} c_{ik}d_{igjk}Y_{igjk} - \sum_{i\in I}\sum_{g\in G}\sum_{j\in J}\sum_{k\in K} t_{ijk}d_{igjk}Y_{igjk} - \sum_{i\in I}\sum_{g\in G}\sum_{k\in K}\left(o_{ik}\frac{\sum_{j\in J} d_{igjk}Y_{igjk}}{Q_{igk}} + \frac{h_{ik}Q_{igk}}{2} + (e_{ik}+a_{ik}Q_{igk})\frac{\sum_{j\in J} d_{igjk}Y_{igjk}}{Q_{igk}}\right) \tag{1}$$

s.t.

$$\sum_{i\in I}\sum_{g\in G} Y_{igjk} \le 1 \quad j\in J,\ k\in K \tag{2}$$

$$\sum_{g\in G} Z_{igk} \le 1 \quad i\in I,\ k\in K \tag{3}$$

$$Y_{igjk} \le Z_{igk} \quad i\in I,\ g\in G,\ j\in J,\ k\in K \tag{4}$$

$$Z_{igk} \le X_i \quad i\in I,\ g\in G,\ k\in K \tag{5}$$

$$X_i \in \{0,1\} \quad i\in I \tag{6}$$

$$Z_{igk} \in \{0,1\} \quad i\in I,\ g\in G,\ k\in K \tag{7}$$

$$Y_{igjk} \in \{0,1\} \quad i\in I,\ g\in G,\ j\in J,\ k\in K \tag{8}$$

$$Q_{igk} \ge 0 \quad i\in I,\ g\in G,\ k\in K. \tag{9}$$

The objective (1) is to maximize the total income minus the total cost, which includes the fixed DC opening costs, the procurement costs, the transportation costs from open DCs to customers and the inventory costs at open DCs. The inventory cost at each DC includes the fixed cost of placing orders, the holding cost of working inventory and the transportation cost. Constraints (2) allow a customer to be assigned to only one facility for each commodity or not to be served. Constraints (3) force each open DC to select only one markup level for each commodity. Constraints (4) and (5) impose that a customer can be assigned to a DC to satisfy her demand for each commodity with a specific markup only if the DC is established and offers the commodity at that markup. Constraints (6), (7) and (8) include the integrality constraints, and constraints (9) enforce non-negativity restrictions on the real-valued variables.

In order for the objective function (1) to be well-defined, the convection $0/0=0$ is considered. Moreover, if commodity k is not offered by DC i with markup level g, i.e., $Z_{igk}=0$, the corresponding inventory cost in the objective becomes zero at optimality; therefore, no constraint is required to guarantee the condition $Z_{igk}=0 \Rightarrow Q_{igk}=0$. In Section 3, it is also shown that whenever $Z_{igk}=0$, its corresponding optimal order-size given in (10) automatically becomes zero by constraints (4). Moreover, recall that when both demand and lead time are constant in a (Q,R) inventory system, the reorder point R can be directly determined based on the order size Q.