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COMPUTATION OF l_1 CENTRAL ESTIMATOR OF LINEAR PARAMETERS

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Abstract

The authors propose a new algorithm for computing l_1 central estimators of linear parameters. A theorem is given saying that such an estimator can be computed by solving a group of linear programming problems with a common feasible region. An overall simplex algorithm for solving such a group of linear programming problems is suggested, which may avoid repeated search of any vertex of the common feasible region. These results may effectively reduce the computation of l_1 central estimators of linear parameters.

Key Words

Parameter estimation, central estimator, unknown but bounded errors, set membership uncertainty, linear programming

1. Introduction

Over the past several years a nonprobabilistic approach has attracted a great deal of attention in the field of parameter estimation, which is usually called the set membership uncertainty (SMU) or unknown but bounded error (UBBE) approach [1–3]. With this approach a central estimator is regarded as an optimal estimator. Its definition can be simply explained as follows. Consider a problem of parameter estimation described by the linear model:

$$y = \Phi\theta^* + \rho$$

where $y \in R^N$ and $\Phi \in R^{N\times n}$ are the known vector and matrix, $\theta^* \in R^n$ is a parameter vector to be estimated, and $\rho \in R^N$ is an unknown error vector. With UBBE approach, ρ is assumed to satisfy the constraint

$$\|\rho\| \le \varepsilon$$

where ε is a given positive number. This inequality ensures that θ^* belongs to the feasible parameter set:

$$\Theta = \{\theta, \theta \in R^n \mid s.t. \|y - \Phi\theta\| \le \varepsilon\}$$

According to this assumption, the following inequality for any n-dimensional vector $\hat{\theta}$ holds:

$$\|\theta^* - \hat{\theta}\| \le \max_{\theta\in\Theta} \|\theta - \hat{\theta}\|$$

In this case, it is natural to determine an estimate of θ^* by minimizing the right side of this inequality. This leads to the definition of a central estimator $\hat{\theta}^c$, that is:

$$\max_{\theta\in\Theta} \|\theta - \hat{\theta}^c\| = \min_{\hat{\theta}\in R^n} \max_{\theta\in\Theta} \|\theta - \hat{\theta}\|$$

In most literature on the UBBE approach, the norm of the model error is taken as l_∞^w, that is:

$$\|\rho\|_\infty^w = \max_{1\le t\le N} w_t|\rho_t|$$

where w_t is a component of the vector $w \in R^N$, which is a given positive weight coefficient,[1] and the norm of the estimation error is mainly taken as l_∞, that is:

$$\|\theta - \hat{\theta}\|_\infty = \max_{1\le i\le n} |\theta_i - \hat{\theta}_i|$$

Under this condition the central estimator $\hat{\theta}^{c\infty}$ and its corresponding maximal estimation error can be computed with the formulas [2]:

$$\hat{\theta}_i^{c\infty} = 0.5 \times (\theta_i^{max} + \theta_i^{min}), \quad i = 1, \cdots, n$$

$$\max_{\theta\in\Theta} \|\theta - \hat{\theta}^{c\infty}\|_\infty = \max_{1\le i\le n} 0.5 \times (\theta_i^{max} - \theta_i^{min})$$

where all θ_i^{max} and θ_i^{min} can be obtained by solving the following $2 \times n$ linear programming problems:

$$\theta_i^{max} = \max_{\theta\in\Theta} \theta_i, \quad \theta_i^{min} = \min_{\theta\in\Theta} \theta_i, \quad i = 1, \cdots, n \qquad (1)$$

Notice that all these linear programmimg problems have a common feasible region.

In recent years, because of the development of l_1 robust control methodologies, l_1 central estimation problem has been considered by some authors [4, 5], which means that the norm of the estimation error is taken as l_1, that is:

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(paper no. 1994-10)

[1] In this work, if a symbol stands for a vector then the same symbol with a subscript will always denote a component of this vector.