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# How does real option value compare with Faustmann value when log prices follow fractional Brownian motion?

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ABSTRACT

Analysis of an extended price series from 1973 to 2016 for New Zealand A grade export logs confirms that the price series is not I(0) stationary. The rejection of the unit root test suggests that it is also not I(1). It is estimated that log prices are fractionally integrated with A grade prices being I(0.78) and the natural logarithm of A grade prices being I(0.83). The implication is that log prices should be modelled using fraction Brownian motion (FBM) rather than geometric Brownian motion (GBM) or as a stationary autoregressive process.

The difference in the NPV calculated using FBM compared to the NPV of classic Faustmann or GBM depends on the fractional difference, log price and volatility. Analysis of the extended A grade price series indicates an H value of about 0.3. At this level the differences at stand age 0, from both Faustmann and GBM, are modest in terms of NPV. However the differences are marked in terms of reserve log price strategy, probability of harvest and rotation age. Differences in NPV between FBM and Faustmann increase and become material as volatility increases.

## 1. Introduction

Forest valuation in New Zealand and many other countries is based on the Faustmann approach (e.g. NZIF, 1999). However, there is increasing interest in recognising the option value inherent in the flexibility that the forest owner has over the timing of harvest. Practitioners have been following research activity in this field and are seeking practical applications.

Research on the use of real option approaches for forest valuation can be differentiated between studies with stationary prices (e.g., Norstrom, 1975; Lohmander, 1988; Brazee and Mendelsohn, 1988; Haight and Holmes, 1991) and those with non-stationary prices (e.g., Clarke and Reed, 1989; Morck et al., 1989; Thomson, 1992; Reed, 1993). In the studies with stationary prices, the expected value of a stand has been found to be higher when stochastic variation in price is exploited, compared to the Faustmann value. However, in studies using non-stationary prices, "there are no gains except when there are fixed costs (e.g., management costs, alternative land uses)"; i.e., the Faustmann value is sufficient (Plantinga, 1998).

Plantinga (1998) also found that "as in previous studies with stationary prices, expected timber values are higher with a reservation price policy compared to the Faustmann model with expected prices. However, the expected values with non-stationary prices are identical to the Faustmann values". Insley (2002) also showed the importance of the underlying stochastic price process in applying a real options approach to forest valuation. She found that "option value and optimal cutting time are significantly different under the mean reversion assumption compared to geometric Brownian motion".

Manley and Niquidet (2010) compared the real option value of a typical New Zealand plantation with the Faustmann value under different log price models. As with the earlier studies, real option value depended heavily on the log price model assumed. Under the assumption that log prices follow a non-stationary random walk with geometric Brownian motion (GBM), real option value was similar to Faustmann values estimated with constant log prices, except when log prices were very low and close to the harvesting cost. However, when log prices follow a mean–reverting random draw or AR(1) process, option value exceeds Faustmann value at all log prices.

Niquidet and Manley (2007) analysed historical log prices in New Zealand and found that virtually all log prices followed a non-stationary process. The analysis indicated that log prices were likely non-stationary random walks (or at least contained a random walk component). A subsequent analysis (reported in Manley, 2013) with an updated time series confirmed this finding. However, a key limitation of this work is the length of the time series (from Q3 1994 to Q1 2011), which limits the size and power of the tests and perhaps more

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importantly the ability to pick up longer term cycles. Although the analysis of Niquidet and Manley (2007) indicated a non-stationary price process for virtually all log grades and regions, tests on a longer time series of A grade export logs for 1973–2001 were inconclusive.

Niquidet and Sun (2012), in reviewing forest product prices, considered 32 publications of which 22 analysed log price or stumpage series. They found conflicting evidence about the price process followed. They noted that research in forest economics had focused on "the two extremes: (1) a nonstationary unit root process which is integrated of order one I(1), and (2) a stationary process integrated of order zero I(0)". Their subsequent analysis of North American lumber and pulp prices led them to reject both stationary and non-stationary null hypotheses. They concluded that lumber and pulp prices are fractionally integrated (i.e. they are I($d$) with $0 < d < 1$) and display long memory. Estimates of the fractional difference parameter (d) ranged between 0.68 and 0.81 for lumber prices and was 0.64 for pulp. Kristoufek and Vosvrda (2014) analysed daily prices between 2000 and 2013 of front futures for 25 commodities. Their implied estimate of d for lumber was 0.86. For other commodities the range was between 0.73 (oats) and 1.2 (copper).

Niquidet and Sun (2012) noted that "in the range $-0.5 < d < 0.5$ time series are stationary and invertible, whereas nonstationary series are those with $d \geq 0.5$. The data-generating process is mean-reverting if $0 \leq d \leq 1$, although for $d \geq 0.5$ the term mean-reversion may be misleading as it only applies to the property of shocks eventually dissipating but the expected mean of the series is undefined as the variance of the series is not finite".

### 1.1. Fractional Brownian motion

Fractional integration is the discrete time counterpart of fractional Brownian motion (FBM). The above suggests that FBM rather than GBM may be more appropriate for modelling log prices over time.

FBM was introduced by Mandelbrot and van Ness (1968) as a generalisation of Brownian motion in which, unlike Brownian motion or GBM, increments may not be independent. It has subsequently been used in a wide range of applications. For example, Baillie (1996) discussed applications of fractional integration in geophysical sciences, macroeconomics, asset pricing models, stock returns, exchange rates and interest rates. Other FBM examples are given by Elliott and Chan (2004) on options and Rostek and Schobel (2013) on financial modelling.

The focus of this paper is to extend the previous work of Manley and Niquidet (2010) to include fractional Brownian motion. Initially we analyse New Zealand log prices including the extended A grade log price series. Having determined that fractional integration or long memory is indicated we then compare real option value under FBM with Faustmann value.

## 2. Approach

### 2.1. Log price analysis

Log prices analysed are:

- Ministry for Primary Industries (MPI) quarterly log prices from Q3 1994[1] to Q3 2016 for each of 9 log grades.
- Weighted average log price (using relative volumes at age 30 for a specified stand as weights).
- A grade price series from Q1 1973 to Q3 2016. This series is a composite series. Data was available for Q1 1973 to Q1 2001 from the now defunct company Fletcher Challenge Forests. We have extended this with MPI data through to Q3 2016. Before doing so we confirmed that there was close alignment of data from the two series over the common period of Q1 1992 to Q2 2001.

[1] Export A and export K price series start in Q1 1992.

Nominal prices are converted to real using CPI. Tests are applied to both untransformed prices and also the natural logarithm of prices. The former are used to allow comparison with previous studies, while the latter are used in the price model adopted.

Tests used are:

- DF-GLS test (Elliott et al., 1996) with null hypothesis of I(1); i.e., that there is a unit root and the series is non-stationary. The lag was selected by the modified Akaike information criterion (MAIC) (Ng and Perron, 2001).
- KPSS test (Kwiatkowski et al., 1992) with null hypothesis of I(0); i.e., that the series is stationary.
- GPH estimate (Geweke and Porter-Hudak, 1983) of fractional differencing parameter ($d$) for any series that has both hypotheses rejected.
- KPSS test on the demeaned fractionally differenced series (Shimotsu, 2006) to confirm whether it is stationary. If a series is I($d$) then its $d$th difference follows a I(0) process.

### 2.2. Price models

Manley and Niquidet (2010) and Manley (2013) used a log price model of a non-stationary random walk with GBM. The discrete-time version of this is that the price at time t ($P_t$) has a lognormal distribution (i.e., $P_t$ is lognormal with $\ln(P_t)$ normally distributed). The model implies that:

$$\ln P_{t+T} = \ln P_t + \mu T - \sigma^2 T/2 + \sum \varepsilon_i \tag{1}$$

where

$P_t$ and $P_{t+T}$ are prices at time t and T years later
μ is expected annual change in log price (expressed as a proportion)
σ is the standard deviation, i.e., the volatility (expressed on an annual basis)
ε is a random normal variable with mean 0 and variance = $\sigma^2$

In contrast, for FBM the log price model is:

$$\ln P_{t+T} = \ln P_t + \mu T - \sigma^2 T^{2H}/2 + \sum W_i^H \tag{2}$$

where

$W_i^H$ is fractional Gaussian noise (FGN) with mean 0, variance = $\sigma^2$ and Hurst coefficient H. The autocorrelations are given by $\rho(T) = ½[(T+1)^{2H} - 2T^{2H} + (T-1)^{2H}]$. The summation extends from $i = 1$ to T.

The Hurst coefficient (H) is related to the fractional difference or memory parameter (d) by the expression $H = d + ½$. Table 1 gives the autocorrelations for different values of T with H varying between 0 and 0.5. In the case where $H = 0.5$, the autocorrelations (for $T > 0$) all become 0 and Model (2) is equivalent to Model (1); i.e., GBM is a special case of FBM with $H = 0.5$.

As a basis for comparison, we also developed a stationary AR(1) model:

$$\ln P_{t+1} = (1-\theta)\,\overline{\ln P} + \theta \ln P_t - \sigma^2/2 + \varepsilon \tag{3}$$

with $\theta = 0.742$.

Examples are provided in Fig. 1 for FBM with $H = 0.001$, $H = 0.3$ and $H = 0.49$ as well as the AR model. Ten different price paths over 50 years, from an initial price of $100/m$^3$, are shown for each price model. They illustrate that as H decreases, prices become more persistent. There is much less of a range when $H = 0.001$ compared to