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Original Research Paper

# Theoretical investigation on heat transfer to burning char of spheroidal particles

Jie Li, Jian Zhang *

*Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China*

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The combustion of biomass fuel provides efficient and inexpensive utilization of solid biomass. Biomass particles generally exhibit non-spherical shapes, which affect the heat and mass transfer and further the char surface reactions. The prolate and oblate spheroidal particles are considered presently. The energy balance is employed for the gas phase near the particle in spheroidal coordinate system. A theoretical study is conducted on the char combustion of the spheroidal particles under either static conditions or forced convection. The expressions for the Nusselt numbers of both prolate and oblate spheroids are obtained. The char combustion rates for the spheroidal particles are affected by the Nusselt number and the particle surface area.



## 1. Introduction

The utilization of solid biomass in industrial combustion apparatus has raised great interest in recent years [1,2]. During combustion processes, biomass particles are consumed by both thermal decomposition reactions and char oxidation. The biomass char combustion is a heterogeneous surface reaction between gas and particle. It is closely related to the instantaneous particle geometry. Electron microscopy images of typical biomass particles show that most of biomass particles are non-spherical [3,4]. For example, straw and pine wood particles approximate to prolate spheroids or cylinders, while sawdust particles tend to be oblate spheroids or discs. The surface area of the non-spherical particle is larger than that of the spherical particle having the same volume. The increasing particle surface area will then affect the combustion dynamics and heat and mass transfer properties [5,6].

The quantitative description on the combustion of non-spherical particles is concerned in developing its mathematical model. Although the complexity and inhomogeneity of non-spherical particles add to the difficulty, there are some studies with focus on the char combustion model for non-spherical particles. Pinho et al. [7] investigated the char combustion for prolate spheroid particles in a quiescent environment. They proposed an implicit expression for the char combustion in the diffusion controlled combustion regime. The char combustion rate for prolate spheroid particles was shown to be greater than that for the spherical particles having the same volume. By means of ellipsoidal coordinates, Grow [8] proposed an integral expression for the oxygen mass flux at the surface of a prolate spheroid under quiescent environment. The ratio of the average mass flux to that for a spherical particle was found to vary from 1.00 to 1.16 as the particle aspect ratio varies from 1.05 to 5. As an extension of Grow's work [8], Gera et al. [9,10] defined an enhancement factor that accounts for the non-spherical shape of biomass particles. It was calculated by solving the ratio of the average oxygen mass flux at the surface of a prolate particle to that for a spherical particle with the same surface area. The enhancement factor is a complex integral expression. Yin et al. [11] further simplified the enhancement factor. A simple algebraic function of the enhancement factor varying with the particle sphericity was proposed. Golovin et al. [12] derived the Peclet numbers for prolate and oblate spheroids and obtained the char combustion rates for spheroidal particles.

It is noted that most of the previous theoretical studies on the char combustion rates for spheroidal particles are based on the mass conservation. The enhancement factors are obtained for the mass transfer between gas and spheroidal particle. On the other hand, the gas-particle heat transfer is also affected by the non-spherical particle shape. The char combustion rates of spheroidal particles can be formulated on the basis of energy conservation. With the aid of spheroidal coordinates, the energy balance in the gas layer near the particle surface is studied by illustrating the

* Corresponding author. Fax: +86 10 62781824.
*E-mail address:* jianzhang@mail.tsinghua.edu.cn (J. Zhang).

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**Nomenclature**

| | |
|---|---|
| $a$, $b$, $c$ | semi-axis of ellipsoid (m) |
| $A$ | particle surface area ($m^2$) |
| $c_p$ | gas specific heat (J/(kg·K)) |
| $E$ | scale factor ($m^3$) |
| $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ | Cartesian coordinate unit vectors (m) |
| $g$ | total gas mass flux (kg/($m^2$·s)) |
| $G_c$ | char burning rate (kg/s) |
| $h$ | gas-particle heat transfer coefficient (W/($m^2$·K)) |
| $H$ | scale factor ($m^{-1}$) |
| $M$ | particle aspect ratio |
| $\mathbf{n}$ | unit vector outward normal to the ellipsoid surface (m) |
| $Nu$ | Nusselt number |
| $Nu_0$ | Nusselt number under static conditions |
| $Pr$ | gas Prandtl number |
| $Q_h$ | oxidation reaction heat of char (kJ/kg) |
| q | total gas heat flux (W/$m^2$) |
| $r$ | sphere radius (m) |
| $Re_k$ | particle Reynolds number |
| $T_k$ | particle temperature (K) |
| $T_\infty$ | gas temperature in surrounding environment (K) |
| $x$, $y$, $z$ | Cartesian coordinates (m) |

*Greek symbols*

| | |
|---|---|
| $\xi$, $\eta$, $\zeta$ | ellipsoidal coordinates ($m^2$) |
| $\lambda$ | gas thermal conductivity (W/(m·K)) |
| $\Pi$ | concentric ellipsoid surface |
| $\Pi_s$ | particle surface |
| $\psi$ | particle sphericity |

*Subscripts*

| | |
|---|---|
| *os* | oblate spheroid |
| *ps* | prolate spheroid |
| *sp* | sphere |
| *sup* | film boundary |

gas heat fluxes associated with the burning char surface. The analytical expressions for the char burning rates of both prolate and oblate spheroids are obtained under static conditions. These expressions are extended to consider the influences of convective heat transfer between gas and particle. The effects of particle aspect ratio on the heat transfer to burning char are delineated.

## 2. Heat transfer to burning char of spheroidal particle

### 2.1. Gas energy conservation near the particle surface

Char reactions with gas phase are assumed to occur on the surface of spheroidal particle. There is no reaction in the gas layer near the particle surface. It means that the single-film approach is adopted under the present conditions of particle size and gas environment temperature [13,14]. No heat is conducted into the particle interior and the particle temperature maintains unchanged. The radiative heat transfer between the surroundings and the particle is ignored. The gas and particle have no relative motion and the flow near the particle is symmetric. The gas temperature is uniform over the particle surface. The gas specific heat is taken as constant. The total gas heat flux at the surface of a concentric spheroid outside the particle is written as

$$q = -\lambda\frac{\partial T}{\partial n} + gc_pT \tag{1}$$

where $T$ and $n$ represent the gas temperature and the outward normal direction at the surface of the concentric spheroid, respectively and $g$ is the total gas mass flux.

The ellipsoidal coordinate system is introduced to study the char combustion of ellipsoidal particles. The surface of an ellipsoid is given by the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Here $a$, $b$, and $c$ denote its semi-axes [15]. The ellipsoidal coordinates ($\xi$, $\eta$, $\zeta$) are obtained by solving the equation $x^2/(a^2+s) + y^2/(b^2+s) + z^2/(c^2+s) = 1$. They can be obtained from the Cartesian coordinates ($x$, $y$, $z$) as follows:

$$\begin{aligned} x^2 &= \frac{(a^2+\xi)(a^2+\eta)(a^2+\zeta)}{(a^2-b^2)(a^2-c^2)} \\ y^2 &= \frac{(b^2+\xi)(b^2+\eta)(b^2+\zeta)}{(b^2-a^2)(b^2-c^2)} \\ z^2 &= \frac{(c^2+\xi)(c^2+\eta)(c^2+\zeta)}{(c^2-a^2)(c^2-b^2)} \end{aligned} \tag{2}$$

The equation $x^2/(a^2+\xi) + y^2/(b^2+\xi) + z^2/(c^2+\xi) = 1$ represents the concentric ellipsoid surface for $\xi$ being constant. It stands for the ellipsoid surface when $\xi = 0$.

The unit vector outward normal to the surface of the concentric ellipsoid is given by

$$\mathbf{n} = \frac{1}{H}\left(\frac{x}{a^2+\xi}\mathbf{e}_x + \frac{y}{b^2+\xi}\mathbf{e}_y + \frac{z}{c^2+\xi}\mathbf{e}_z\right) \tag{3}$$

where

$$H = \left[\left(\frac{x}{a^2+\xi}\right)^2 + \left(\frac{y}{b^2+\xi}\right)^2 + \left(\frac{z}{c^2+\xi}\right)^2\right]^{1/2} \tag{4}$$

Thus the normal derivative of gas temperature can be written as

$$\frac{\partial T}{\partial n} = \nabla T\cdot\mathbf{n} = \frac{1}{H}\left(\frac{\partial T}{\partial x}\frac{x}{a^2+\xi} + \frac{\partial T}{\partial y}\frac{y}{b^2+\xi} + \frac{\partial T}{\partial z}\frac{z}{c^2+\xi}\right) \tag{5}$$

Utilizing the results of partial differentiation to Eq. (2) yields

$$\frac{\partial T}{\partial n} = \frac{2}{H}\frac{\partial T}{\partial \xi} \tag{6}$$

Thus Eq. (1) becomes

$$q = -\frac{2\lambda}{H}\frac{\partial T}{\partial \xi} + gc_pT \tag{7}$$

Integrating the above equation over the surface of the concentric ellipsoid gives

$$\iint_{\Pi} q ds = -\iint_{\Pi}\frac{2\lambda}{H}\frac{\partial T}{\partial \xi}ds + \iint_{\Pi} gc_pTds \tag{8}$$

where $\Pi$ denotes the concentric ellipsoid surface. The gas temperature is only a function of the coordinate $\xi$ by symmetry. Thus the above integrations are obtained as

$$\iint_{\Pi} gc_pTds = G_cc_pT \tag{9}$$

$$\iint_{\Pi}\frac{2\lambda}{H}\frac{\partial T}{\partial \xi}ds = 8\pi\lambda(a^2+\xi)^{1/2}(b^2+\xi)^{1/2}(c^2+\xi)^{1/2}\frac{\partial T}{\partial \xi} \tag{10}$$

where $G_c$ is the total mass flow rate on the particle surface or the char burning rate. Utilizing the gas energy conservation and the temperature boundary conditions at the particle surface, we have

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