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# The effect of the flame phase on thermoacoustic instabilities

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ABSTRACT

This paper concerns the influence of the phase of the heat release response on thermoacoustic systems. We focus on one pair of degenerate azimuthal acoustic modes, with frequency $\omega_0$. The same results apply for an axial acoustic mode. We show how the value $\phi_0$ and the slope $-\tau$ of the flame phase at the frequency $\omega_0$ affects the boundary of stability, the frequency and amplitude of oscillation, and the phase $\phi_{qp}$ between heat release rate and acoustic pressure. This effect depends on $\phi_0$ and on the nondimensional number $\tau\omega_0$, which can be quickly calculated. We find for example that systems with large values of $\tau\omega_0$ are more prone to oscillate, i.e. they are more likely to have larger growth rates, and that at very large values of $\tau\omega_0$ the value $\phi_0$ of the flame phase at $\omega_0$ does not play a role in determining the system's stability. Moreover for a fixed flame gain, a flame whose phase changes rapidly with frequency is more likely to excite an acoustic mode.

We propose ranges for typical values of nondimensional acoustic damping rates, frequency shifts and growth rates based on a literature review. We study the system in the nonlinear regime by applying the method of averaging and of multiple scales. We show how to account in the time domain for a varying frequency of oscillation as a function of amplitude, and validate these results with extensive numerical simulations for the parameters in the proposed ranges. We show that the frequency of oscillation $\omega_B$ and the flame phase $\phi_{qp}$ at the limit cycle match the respective values on the boundary of stability. We find good agreement between the model and thermoacoustic experiments, both in terms of the ratio $\omega_B/\omega_0$ and of the phase $\phi_{qp}$, and provide an interpretation of the transition between different thermoacoustic states of an experiment. We discuss the effect of neglecting the component of heat release rate not in phase with the pressure $p$ as assumed in previous studies. We show that this component should not be neglected when making a prediction of the system's stability and amplitudes, but we present some evidence that it may be neglected when identifying a system that is unstable and is already oscillating



## 1. Introduction

We first review fundamentals of thermoacoustic instabilities in Section 1.1 and present three key questions on the subject, then review the existing literature in Section 1.2, and briefly outline the paper in Section 1.3.

### 1.1. Motivation of this work

Rayleigh [1] was the first to observe that if part of the fluctuating heat release rate $q$ is in phase with the acoustic pressure $p$ self sustained acoustic oscillations can occur. Accounting also for acoustic losses [2,3], considering the case of a single acoustically compact flame and assuming a low Mach number flow, the criterion requires that

$$\frac{1}{T}\int_t^{t+T} q(t)p(t)dt > \quad \text{acoustic losses} \tag{1}$$

where $T = 2\pi/\omega$ is the period of the thermoacoustic instability and $\omega$ its angular frequency, $q$ and $p$ are considered at the flame location, and we assume thermodynamic equilibrium and a perfect gas. Under suitable assumptions discussed later, one can express[1] the fluctuating heat release rate as function of the pressure $p$ as $q = \mathcal{Q}[p]$. For the sake of brevity, in the following we will often refer to $q$ as the flame response to the pressure $p$, or simply as the flame response. We assume and substitute a sinusoidal pressure

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[1] With the exception of the trivial cases where the flame is located at a pressure node of the acoustic field at frequency $\omega$. These cases cannot be unstable because the left hand side of (1) is zero.

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**Nomenclature**

| | |
|---|---|
| $'$ | the prime denotes time derivative of the preceding quantity |
| $\hat{}$ | the hat denotes the Fourier transform of the underlying quantity |
| $u$ | acoustic velocity in the azimuthal direction, suitably nondimensionalised |
| $u_{ax}$ | acoustic velocity in the axial direction, typically long the axis of the burner, suitably nondimensionalised |
| $p$ | acoustic pressure, suitably nondimensionalized |
| $q$ | fluctuating heat release rate, suitably nondimensionalised, often called flame response |
| $Q(A, \omega)$ | describing function of the fluctuating heat release rate $q = q[p]$ as function of $p$. Defined in (3) |
| $n$ | azimuthal order of the mode, e.g. $n = 3$ refers to the third azimuthal |
| $A_j$ | slowly varying amplitudes of oscillations, introduced in (29) |
| $\alpha$ | equivalent acoustic damping coefficient, appearing in (7b) |
| $\beta$ | flame strength, i.e. the nondimensional linear flame response gain as function of $p$, as in $\|q\| \propto \beta \|p\|$ |
| $\gamma_k$ | standard deviation of the $k$-th time delay, see (12), appearing also in Fig. 4 |
| $\delta$ | nonlinear saturation coeff. as in (37a) |
| $\eta_j n$ | amplitudes of the azimuthal acoustic velocity of the 2 modes as in (8), for $j = 1, 2$ |
| $\eta'_j$ | amplitudes of the acoustic pressure of the 2 modes as in (8), for $j = 1, 2$ |
| $\theta$ | azimuthal coordinate along the annular combustion chamber, $\theta \in [0, 2\pi)$ |
| $\kappa$ | nonlinear saturation coefficient, appearing in (16) |
| $\lambda$ | eigenvalue, $\lambda = \sigma + i\omega$, with $\omega$ in rad/s |
| $\mu$ | L2 norm of the mode, as defined in (11) |
| $\nu$ | expression for the growth rate appearing in (38) |
| $\sigma$ | growth rate, i.e. real part of the eigenvalue $\lambda = \sigma + i\omega$ |
| $\tau$ | equivalent time delay of the transfer function $\hat{q}/\hat{p}$ as introduced in (15), i.e. minus the local slope of of the flame phase of such transfer function at frequencies close to $\omega_0$. |
| $-\tau\omega_0$ | nondimensional slope of the flame phase in the vicinity of the acoustic mode with frequency $\omega_0$ |
| $\phi(\omega)$ | flame phase response, i.e. the argument of $Q$, as function of the frequency $\omega$. We assume that it does not depend on the amplitude of oscillation $A$. This quantity depends on the geometry upstream of the flame and on the flame response. |
| $\phi_0$ | flame phase at the acoustic frequency $\omega = \omega_0$, i.e. $\phi(\omega_0)$ |
| $\phi_{qp}$ | phase between $q$ and $p$ of a thermoacoustic mode at frequency $\omega_B$, i.e. $\phi(\omega_B)$ |
| $\varphi_j$ | slowly varying phase of the $j$-th azimuthal mode, $j = 1, 2$ |
| $\varphi$ | slowly varying phase difference $\varphi_1 - \varphi_2$ of the two azimuthal mode |
| $\chi$ | radial and axial shape of the azimuthal modes, $\chi(r, z)$ |
| $\omega$ | angular frequency, variable |
| $\omega_0$ | angular *acoustic* frequency of oscillation when the flame and the damping are virtually shut off and the system becomes conservative. This is the frequency of oscillation of the acoustic mode without being excited by the flame and without being damped by the acoustic losses |
| $\omega_B$ | angular *thermoacoustic* frequency of the system if the flame response gain $\beta$ is virtually decreased until the system is neutrally stable, i.e. the growth rate $\sigma$ becomes zero, solution of (27). We prove that $\omega_B$ is also the frequency of the limit-cycle solution if the flame phase does not depend on the amplitude and the damping losses are linear, as is the case in this work |
| $\omega_{LC}$ | angular frequency of oscillation at the limit cycle, proved to match $\omega_B$ |
| $\Omega$ | domain of the combustor |

$p(t) = A\cos(\omega t)$ in (1):

$$\frac{1}{T}\int_t^{t+T} \mathcal{Q}[A\cos(\omega t)]A\cos(\omega t)dt > \quad \text{acoustic losses} \tag{2}$$

We now define the describing function $Q(A, \omega)$ of an operator $\mathcal{Q}$ of a sinusoidal input at frequency $\omega$ and with an amplitude $A$ similarly to [4]:

$$Q(A, \omega) \equiv \frac{1}{A}\frac{2}{T}\int_t^{t+T} \mathcal{Q}[A\cos(\omega t)]e^{-i\omega t}dt \tag{3}$$

We multiply and divide the left hand side of (2) by $2/A^2$, and by substituting the real part of (3) we obtain

$$\frac{1}{2}\mathrm{Re}[Q(A, \omega)]A^2 > \quad \text{acoustic losses} \tag{4}$$

On the left hand side, we recover the typical structure of a conservative potential; for example, for a linear spring with constant $k$ loaded with a steady displacement $A$, the energy is $kA^2/2$, where the describing function is real valued, does not depend on the amplitude $A$ because the spring is linear and matches the constant $k$. We can rewrite the complex valued describing function in terms of its real valued, non-negative gain $G$ and real valued phase response $\phi$:

$$Q(A, \omega) = G(A, \omega)e^{i\phi(A,\omega)} \tag{5}$$

In the following we will refer for brevity to $G$ as flame gain and to $\phi$ as flame phase. By substituting (5) in (1) we obtain:

$$\frac{1}{2}G(A, \omega)\cos(\phi(A, \omega))A^2 > \quad \text{acoustic losses} \tag{6}$$

Eq. (6) allows the same interpretation of (1), but in terms of the flame gain $G$ and flame phase $\phi$ of the describing function $Q$. We then review known results discussed first by [1]. We observe that the acoustic loss term on the right hand side of (6) is positive, so that in order for (4) to hold we require that $\cos(\phi) > 0$, i.e. that $-\pi/2 < \phi(A, \omega) < \pi/2$. Once this first necessary condition is satisfied, there exists a threshold value of the gain above which (4) is verified and a thermoacoustic oscillation ensues.

This perspective in terms of an acoustic energy balance correctly captures the dominant feature of the thermoacoustic problem as a self excited closed loop system, which in an enclosed cavity has a set of countable thermoacoustic eigenmodes. We can interpret the Rayleigh criterion at the frequency $\omega$ of the nonlinearly saturated eigenmode at a limit cycle, i.e. at the dominant frequency peak of a thermoacoustically unstable experiment. We distinguish $\omega$ from the eigenfrequency $\omega_0$ of the acoustic mode of the combustor obtained when a passive flame is considered. We now consider three other scenarios, where the Rayleigh criterion does not allow us to conclude much.