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Discrete Optimization

# A computational approach for eliminating error in the solution of the location set covering problem

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ABSTRACT

The location set covering problem continues to be an important and challenging spatial optimization problem. The range of practical planning applications underscores its importance, spanning fire station siting, warning siren positioning, security monitoring and nature reserve design, to name but a few. It is challenging on a number of fronts. First, it can be difficult to solve for medium to large size problem instances, which are often encountered in combination with geographic information systems (GIS) based analysis. Second, the need to cover a region efficiently often brings about complications associated with the abstraction of geographic space. Representation as points can lead to significant gaps in actual coverage, whereas representation as polygons can result in a substantial overestimate of facilities needed. Computational complexity along with spatial abstraction sensitivity combine to make advances in solving this problem much needed. To this end, a solution framework for ensuring complete coverage of a region with a minimum number of facilities is proposed that eliminates potential error. Applications to emergency warning siren and fire station siting are presented to demonstrate the effectiveness of the developed approach. The approach can be applied to convex, non-convex and non-contiguous regions and is unaffected by arbitrary initial spatial representations of space.



## 1. Introduction

An important spatial optimization problem involves determining the fewest number of facilities and where those facilities should be placed in order to suitably cover regional demand for service. Facilities to be sited or evaluated have included fire stations, emergency warning sirens, weather radar equipment, rain gauges, cellular platforms, airports and transit stops, among others. The facility provides a service that may be characterized in a spatial sense, either in terms of distance or travel time, and there is interest in ensuring people/demand are suitably served. For example, suitable service provided by an emergency warning siren is often defined as people being within a 70 db audible range of a siren so that they may be alerted to an emergency situation (i.e., severe weather, toxic spill, nuclear release, etc.) when it occurs. The goal in many cases is therefore to cover a region with demand distributed throughout (Aly and White, 1978; Benveniste, 1982; Murray and O'Kelly, 2002; Alexandris and Giannikos, 2010).

This problem is widely recognized as a set covering problem, or a Location Set Covering Problem (LSCP) in a geographic setting. Initially detailed by Berge (1957) and formulated in Fulkerson and Ryser (1961) as well as Edmonds (1962), much interest and research has been devoted to applying, solving and extending the set covering problem (Church and Murray, 2013). Toregas et al. (1971) were the first to discuss and apply the LSCP, emphasizing its applicability for fire station siting as well as school and library placement. In the years that have followed, the LSCP has been broadly applied because it can be adapted to reflect the goals and objectives of many planning situations. As a result, Murray et al. (2010) note that it is one of the most highly cited location modeling approaches.

Over the last 10 years, with the proliferation of geographic information systems (GIS) and detailed spatial information, work on the LSCP has come to recognize potential limitations and/or issues with its application. In particular, Murray and O'Kelly (2002) illustrated the potential for significant error when demand points are used to represent geographic space. Under certain conditions this error translates to an under-estimation of the true minimum number of facilities necessary to completely cover, or serve, a region. On the other hand, Murray et al. (2008) demonstrated another facet of error in attempting to cover a region when demand polygons are used to represent geographic space. Under certain conditions this particular error is an over-estimation of the true minimum number of facilities needed to serve a region because each demand polygon needs to be completely covered by at least one facility and partial coverage of demand polygons are ignored. Therefore, if the intent is to completely cover a region, the use of either points or polygons to represent geographic space

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can be problematic as this could translate into thousands or millions of dollars in wasteful expenditures to build and maintain facilities due to inefficient spatial configuration.

The intent of this paper is to use the under- and over-estimation information to establish valid bounds on the true minimum number of facilities. With this, an approach for solving the problem is developed to iteratively improve established bounds on the true minimum, offering the potential for eliminating any error. The next section provides a background review of the LSCP. This is followed by a formal specification of the problem. Theoretical and computational properties of the LSCP associated with geographic representation are given. Application results are presented to demonstrate these properties. The paper ends with discussion and conclusions.

## 2. Background

Extensive reviews of coverage models may be found in the literature, including that of Schilling et al. (1993), Owen and Daskin (1998), ReVelle and Eiselt (2005) and Church and Murray (2013). As noted previously, Toregas et al. (1971) were the first to structure and apply the LSCP. The demand for service is assumed in Toregas et al. (1971) to be at known points, or "user" nodes, and the intent is to site a minimum number of facilities among a discrete set of potential points in order to serve all user demand within an established response time or distance standard, *s*.

Aly and White (1978) and Benveniste (1982) detail that the intent of service structured in the LSCP is to cover an area/region, not simply a set of representative points. They highlight that errors may occur as a result of using points as a simplification of a region, as points will get covered but not necessarily the geographic space between points. Aly and White (1978) further note that error is also likely when the centroids of different polygon unit configurations are considered. Though focused on the issue of point aggregation in applying the LSCP, the work of Daskin et al. (1989) and Current and Schilling (1990) also recognized potential errors that could result in the use of points. In particular, Daskin et al. (1989) observed that errors are possible through the process of converting continuous demand or demand zones into node based demand.

While there is a tradition of using points of demand in location modeling (see Miller, 1996; Church, 1999), Murray and O'Kelly (2002) demonstrated for the LSCP that the use of points is problematic and likely introduces significant error in obtained results. Specifically, points will be served/covered as required by the LSCP, but not necessarily the geographic space between points, resulting in service coverage gaps. This work confirmed the observations made by Aly and White (1978) and others about error in applying the LSCP when points are used to simplify spatial units or approximate a continuous region. Murray and O'Kelly (2002) concluded that insufficient spatial coverage by sited facilities renders too few facilities being sited. An alternative modeling strategy is to use polygons of demand, and require total coverage of each demand polygon. However, Murray et al. (2008) highlighted a different type of possible error, illustrating the sensitivity to demand unit configuration hypothesized in Aly and White (1978). Specifically, Murray et al. (2008) showed that the use of polygons to represent regional demand resulted in too many facilities being located. Whether demand points or polygons are utilized, there is inefficiency, yet the intent of using the model, the LSCP, is to seek out system efficiencies. When too few facilities are sited using demand points, the system configuration is inefficient and more facilities will eventually be needed to cover the entire region. Such an incremental addition is typically sub-optimal as it violates the 3$^{rd}$ law of location science (Church and Murray, 2009): "sites of an optimal multisite pattern must be selected simultaneously rather than independently, one at a time." Alternatively, when too many facilities are sited using demand polygons, inefficiency is due to over-expenditure, and is further magnified by cumulative annual operational costs. Murray (2005) suggests that the above errors are related to the modifiable areal unit problem (MAUP), where obtained analytical findings are sensitive to scale and unit definition.

A problem of interest to Kershner (1939) was determining the minimum number of circles with a given radius to cover a rectangle. This can be conceived of as an LSCP, where a circle represents facility coverage. Kershner (1939) approached the problem in an aspatial manner, focusing on establishing upper and lower bounds on the minimum number without concern for where the circles should be positioned. The intuition behind this is associated with the density of circles, where overlap is the least when the number of circles is as small as possible. There have been various attempts to improve upon these bounds, and extend them to higher ordered convex polygons (e.g., 5-gon and 6-gon), with Gruber (1998) representing one of the more recent. Murray et al. (2008) note that these theoretical bounds are valid for a non-convex polygon, but conclude that the bounds are not very tight, rendering the information of limited value in actual planning, not to mention the fact that where to place them is not addressed.

A final related area of work associated with objective solution bounds for the LSCP is the p-center problem. Hakimi (1964) was among the first to describe this problem, with subsequent formal specification by Minieka (1970), among others. The p-center problem involves siting *p* facilities so as to minimize the maximum distance user demand is from its closest facility. The continuous space version of the p-center problem for serving a region was studied by Suzuki and Okabe (1995), Suzuki and Drezner (1996) and Wei et al. (2006). Of significance here is the relationship between the LSCP and the p-center problem noted by Minieka (1970), and the subsequent exploitation of this relationship to devise an iterative approach that solved an LSCP to provide a bound on the optimum p-center solution. It is not surprising that the converse is true as well. Murray et al. (2008) discuss that for particular problem characteristics, solution of the p-center problem can provide a valid upper bound for the LSCP.

## 3. Problem specification

The review in the previous section has detailed that the LSCP is an important optimization model, and there continues to be a need for enhanced solution of this problem. Of course this need goes beyond computational considerations as it is fundamentally essential to understand any potential error and eliminate it if at all possible.

To begin, a formal definition of the problem is:

*Location Set Covering Problem (LSCP)*

Determine the fewest number of facilities and where those facilities should be placed in order to suitably cover regional demand for service, where demand is represented as discrete objects (e.g., points, lines or polygons) and facilities are limited to an finite set of potential locations.

A mathematical statement of the LSCP relies on the following notation:

$i$ = index of demand objects to be served (entire set $I$);

$j$ = index of potential facility locations (entire set $J$);

$\Phi$ = region to be served;

$a_{ij} = 1$ if potential facility $j$ can suitably serve demand $i$, 0 otherwise;

$\Omega_i = \{j | a_{ij} = 1\}$.

As approached by many in the literature, including Toregas et al. (1971), the response time or distance standard, *s*, is often utilized