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Multi-objective optimized fuzzy-PID controllers for fourth order nonlinear systems

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ABSTRACT

In this paper, the Multi-objective Genetic Algorithm (MOGA) is used to obtain the Pareto frontiers of conflicting objective functions for the fuzzy-Proportional-Integral-Derivative (fuzzy-PID) controllers. The ball–beam and inverted pendulum fourth order nonlinear systems are regarded as nonlinear benchmarks. The considered objective functions for the ball–beam system are the distance error of the ball, the angle error of the beam, and the control effort. For the inverted pendulum system, the objective functions are the distance error of the cart, the angle error of the pendulum, and the control effort, which must be minimized simultaneously. The Pareto fronts are compared with those obtained by Multi-objective Particle Swarm Optimization (MOPSO). Four points are chosen from nondominated solutions of the obtained Pareto fronts based on the three conflicting objective functions and used for illustration of the state variables of the controlled systems. Obtained results elucidate the efficiency of the proposed controller in order to control nonlinear systems.

1. Introduction

Zadeh originally proposed the fuzzy logic and the fuzzy set theory [1,2]. Fuzzy systems are knowledge-based or rule-based systems formed via human knowledge and heuristics. They have been applied for a wide range of researching fields, such as control, communication, medicine, management, business, psychology, etc. The most significant applications and studies about fuzzy systems have concentrated on the control area [3–10]. The development of fuzzy-PID controllers for various engineering problems has been a major research activity in recent years. Duan et al. proposed an inherent saturation of the fuzzy-PID controller revealed due to the finite fuzzy rules [11]. Karasakal et al. applied fuzzy PID controllers based on an online tuning method and rule weighing in [12]. Boubertakh et al. proposed new auto-tuning fuzzy PD and PI controllers using reinforcement-learning algorithm for single-input single-output and two-input two-output systems [13]. In this way, the heuristic parameters of fuzzy-PID controllers have to be determined via an appropriate approach. A very effective way to choose these parameters is the use of evolutionary algorithms [14], such as the Genetic Algorithm (GA) [15] and particle swarm optimization (PSO) [16], etc. In [17], a constrained optimization of a simple fuzzy-PID system was designed for the online improvement of PID control performance during productive control runs. Oh et al. developed a design methodology for a fuzzy PD cascade controller for a ball–beam system using particle swarm optimization (PSO) [18]. Mahmoodabadi et al. designed fuzzy controllers for nonlinear systems using MOPSO based on the Lorenz dominance method [19]. Sahib proposed a type of controller consisting of proportional, integral, derivative, and second order derivative terms optimized using the PSO algorithm for an automatic voltage regulator system [20].

In this paper, a novel optimal fuzzy-PID control strategy is proposed and implemented on two nonlinear benchmark systems. Governing equations for ball–beam and inverted pendulum systems transformed to the state-space forms. Two fuzzy inference engines are utilized. Due to having some different objective functions, MOGA and MOPSO are applied and three and two dimensional Pareto front figures are shown. The conflicting objective functions for ball–beam system are the distance error of the ball, the angle error of the beam, and the control effort. For inverted pendulum system, those are the distance error of the cart, the angle error of the pendulum, and the control effort. The simulation results corresponding to the optimum points demonstrate that the designed controller has the superior performance in comparison with reported results in published literature.

The rest of this paper is organized as follows. Section 2 gives a brief description on the fuzzy-PID controller. Section 3 presents the multi-objective optimization genetic algorithm. In Section 4, the

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dynamic models of the ball–beam and inverted pendulum systems are recalled. Furthermore, their optimal fuzzy-PID controllers, simulation results and comparison studies to verify the capability of the proposed controller are shown in this section. Finally, Section 5 concludes the paper.

2. Fuzzy-PID controller

The PID controller has a long history in the control engineering and is accepted in a lot of real applications due to the simple structure. Hence, this controller is widely used still in so many industrial applications despite offering several new techniques. Consider a fourth order nonlinear system with Equation (1).

$$\ddot{x} = f_1 + b_1 F$$
$$\ddot{\theta} = f_2 + b_2 F \tag{1}$$

where f_1, f_2, b_1, and b_2 are nonlinear functions and F is the control input. The state-space formulation of the system can be written as Equation (2).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = f_1 + b_1 F \tag{2}$$
$$\dot{x}_4 = x_5$$
$$\dot{x}_5 = x_6$$
$$\dot{x}_6 = f_2 + b_2 F$$

where $x = [x_1, x_2, x_3, x_4, x_5, x_6] = \left[\int x, x, \dot{x}, \int\theta, \theta, \dot{\theta}\right]$ is the state vector with desired value $x^d = [x_1^d, x_2^d, x_3^d, x_4^d, x_5^d, x_6^d] = \left[\int x^d, x^d, \dot{x}^d, \int\theta^d, \theta^d, \dot{\theta}^d\right]$. The PID controller with inputs $e_x(t) = x - x^d$ and $e_\theta(t) = \theta - \theta^d$ and output $F_{pid}(t)$ is commonly defined as Equation (3).

$$\begin{aligned} F_{pid} &= K_{px} e_x(t) + K_{ix}\int_0^t e_x(\tau)d\tau + K_{dx}\frac{de_x(t)}{dt} \\ &\quad + K_{p\theta} e_\theta(t) + K_{i\theta}\int_0^t e_\theta(\tau)d\tau + K_{d\theta}\frac{de_\theta(t)}{dt} \end{aligned} \tag{3}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative gains, respectively. The adjustment and determination of these design parameters are key issues to design PID controllers. Hence, the fuzzy logic approach is applied to calculate the gains adaptively.

$$F_{Fuzzy\ pid} = \hat{K}_{ix} f_1 + \hat{K}_{px} f_2 + \hat{K}_{dx} f_3 + \hat{K}_{i\theta} f_4 + \hat{K}_{p\theta} f_5 + \hat{K}_{d\theta} f_6 \tag{4}$$

where $F_{Fuzzy\ pid}$ is the fuzzy-PID control action. $f_i, i = 1, 2, \ldots, 6$ are the fuzzy variables with inputs $\int xdt, x, \frac{dx}{dt}, \int\theta dt, \theta$ and $\frac{d\theta}{dt}$, respectively, and should be obtained by Single Input Fuzzy Inference Motor (SIFIM). Furthermore, in Equation (4), the variables $\hat{K}_{ix}, \hat{K}_{px}, \hat{K}_{dx}, \hat{K}_{i\theta}, \hat{K}_{p\theta}$ and $\hat{K}_{d\theta}$ are calculated by Equation (5).

$$\hat{K}_{ix} = K_{ix}^b + K_{ix}^r \Delta W_1$$
$$\hat{K}_x = K_x^b + K_x^r \Delta W_2$$
$$\hat{K}_{dx} = K_{dx}^b + K_{dx}^r \Delta W_3$$
$$\hat{K}_{i\theta} = K_{i\theta}^b + K_{i\theta}^r \Delta W_4$$
$$\hat{K}_\theta = K_\theta^b + K_\theta^r \Delta W_5$$
$$\hat{K}_{d\theta} = K_{d\theta}^b + K_{d\theta}^r \Delta W_6 \tag{5}$$

where $\Delta W_i, i = 1, 2, \ldots, 6$ are the fuzzy variables with inputs $\int xdt, x, \frac{dx}{dt}, \int\theta dt, \theta$ and $\frac{d\theta}{dt}$, respectively, and should be obtained by Preferrer Fuzzy Inference Motor (PFIM). $K_{ix}^b, K_x^b, K_{dx}^b, K_{i\theta}^b, K_\theta^b$ and $K_{d\theta}^b$ are the base variables and $K_{ix}^r, K_x^r, K_{dx}^r, K_{i\theta}^r, K_\theta^r$ and $K_{d\theta}^r$ are the regulation variables. The base and regulation variables can be obtained by the try and error process. However, one of the best solutions to find these to have an optimal controller is the use of optimization approaches such as evolutionary methods, such as the genetic algorithm.

3. Optimization

The genetic algorithm is an approach for solving optimization problems based on biological evolution via modification of a population of individual solutions, repeatedly. At each level, individuals are chosen randomly from the current population (as parents) then employed to produce the children for the next generation. In this paper, toolbox optimization of MATLAB (R2012a) with the following operators is implemented for optimal design of the fuzzy-PID controllers.

3.1. Population size

Increasing the population size enables the genetic algorithm to search more points and thereby obtain a better result. However, the larger the population size, the longer it takes for genetic algorithm to compute each generation.

3.2. Crossover options

Crossover options specify how the genetic algorithm combines two individuals, or parents, to form a crossover child for the next generation.

3.3. Crossover fraction

The crossover fraction specifies the fraction of each population, other than elite children, that is made up of crossover children.

3.4. Selection function

Selection options specify how the genetic algorithm chooses parents for the next generation.

3.5. Migration options

Migration options specify how individuals move between subpopulations. Migration occurs if you set population size to be a vector of length greater than 1. When migration occurs, the best individuals from one subpopulation replace the worst individuals in another subpopulation. Individuals that migrate from one subpopulation to another are copied. They are not removed from the source subpopulation.

3.6. Stopping criteria options

Stopping criteria determine what causes the algorithm to terminate.

In this paper, the configuration of the genetic algorithm is set as the values given in Table 1.

Furthermore, the multi-objective optimization of the proposed fuzzy-PID controller would be done with respect to twelve design variables and three objective functions. The base values [$K_{ix}^b, K_{px}^b, K_{dx}^b, K_{i\theta}^b, K_{p\theta}^b, K_{d\theta}^b$] and regulation values [$K_{ix}^r, K_{px}^r, K_{dx}^r, K_{i\theta}^r, K_{p\theta}^r, K_{d\theta}^r$] are