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Full Length Article

A theoretical and experimental study on geometric nonlinearity of initially curved cantilever beams

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a theoretical and experimental study on large deflection behavior of initially curved cantilever beams subjected to various types of loadings. The physical system as a straight cantilever beam subjected to a tip concentrated load is considered in this study. Nonlinear differential equations are obtained for large deflection analysis of such a straight cantilever beam, and this problem is known to involve geometrical nonlinearity. The equations are solved numerically with the help of MATLAB® computational platform to get deflection profiles of the concerned problem. These results are imposed subsequently on the center line of an initially curved beam to get theoretical load-deflection behavior of curved beam problems. To verify the theoretical model, experiment is carried out with the master leaf of a leaf spring bundle by modeling it as an initially curved cantilever beam. The effects of initial clamping and geometry variations in the eye-region are observed from experimental investigation which is commonly neglected in the mathematical formulation. Comparisons of the theoretical results with the experimental results are quite good, but the avenues for further improvement are also reported. The proposed approach is further extended to study large deflection behavior of an initially curved cantilever beam subjected to distributed and combined load. These results are successfully validated with existing results for straight beams and some new results are furnished for initially curved cantilever beams.

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1. Introduction

In structural analysis, two types of nonlinearities are most commonly encountered: geometric and material. Material nonlinearity is associated with nonlinear stress–strain relations whereas nonlinear curvature–slope and strain–displacement relations give rise to geometric nonlinearity. Depending on the nature of the problem any one or both of the nonlinearities are included in the analysis. In the earlier years, studies of deflection behavior of a cantilever beam under different loadings were based on linear models in order to simplify the analysis. Several researchers [1–3] pointed out that for better characterization of such beams, analysis should be carried out through geometric non-linear model.

Geometrically nonlinear large deflection problem of elastic cantilever beam under tip concentrated vertical load had been solved classically by Bisshopp and Drucker [1], and afterward many researchers have extended the theory. Wang [2,3] proposed a simple numerical method for analyzing nonlinear bending of beam under tip concentrated and uniformly distributed loads respectively. Beléndez et al. [4,5] also studied the same problem, both theoretically and experimentally. Kumar et al. [6] suggested genetic algorithm based search strategies in the context of direct numerical solution of governing differential equation and the principle of stationarity of the energy functional in the equilibrium state. Dado and Al-sadder [7] developed an approach that approximates the angle of rotation by a polynomial function and applied this method effectively for complex load on non-prismatic beam with very large deflection. Banerjee et al. [8] proposed non-linear shooting and Adomian decomposition methods to determine the large deflection of a cantilever beam under arbitrary loading conditions. Chen [9] proposed an integral approach for large deflection study of a cantilever beam with complex load and varying beam properties. Roy and Saha [10] applied a geometrically updating technique by using variational method to find out deflection profiles of non-uniform beams under various loading conditions. Large deflection of beams made of functionally graded material had been studied by Almeida et al. [11] using a tailored Lagrangian formulation and also by several other researchers [12,13]. Xiao-Ting He et al. [14] proposed a new perturbation method with two small parameters, describing the effect of load and geometry of the problem, to solve nonlinear large deflection problem of initially curved beams under two different boundary conditions. Large deflection problem of initially straight

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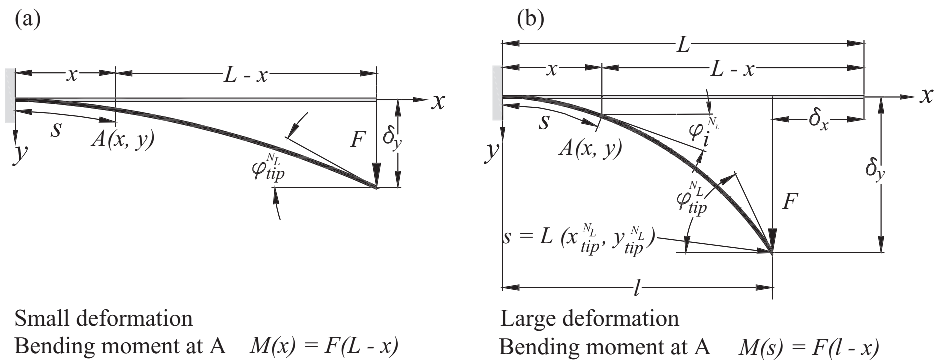

Fig. 1. (a) Small deformation and (b) large deformation of a cantilever beam.

cantilever beam under follower type loading have been solved numerically by several researchers [15,16]. Shvartsman [17] studied large deflection of a curved cantilever beam under follower force by direct numerical method, whereas Nallathambi et al. [18] studied the same problem for a constant curvature cantilever beam by fourth order R-K method.

Design and manufacture of automotive leaf spring using functionally graded and composite materials have been addressed by several researchers [19–23]. Shenhua et al. [24] carried out experimental work on precision roll-forging taper-leaf spring of vehicle, and results have been used in the design of roll-forging process and dies for the forming of taper-leaf springs. Osipenko et al. [25] introduced a contact problem in the theory of leaf spring bending. Sugiyama et al. [26] reported development of nonlinear elastic leaf spring model for multi-body vehicle system. Rahman et al. [27] carried out nonlinear geometric analysis of parabolic leaf spring. Charde et al. [28] used strain gauge technique to evaluate the stress field in the master leaf of a leaf spring and compared the results with finite element method.

Large deflection study of an initially straight cantilever beam under different loading is ever interesting and a huge number of studies are reported in the literature. However, geometric nonlinear analyses of an initially curved cantilever beam under different loading conditions are few. The present paper focuses on both theoretically and experimentally geometric nonlinear behavior of an initially curved cantilever beam under different loading conditions. For the purpose of experimentation, the master leaf of a leaf spring bundle is considered as a cantilever beam with initial curvature.

2. Mathematical formulation

Large deflection problem of cantilever beams is generally analyzed in curvilinear coordinate system. Euler Bernoulli beam theory in curvilinear coordinate system (s, n) is $1/\rho = M/EI$ [1], where curvature $1/\rho = d\varphi/ds$. So Euler Bernoulli bending moment–curvature relationship is given as follows,

$$EI\frac{d\varphi}{ds} = M \tag{1}$$

In equation (1), φ is the slope dy/dx at location s, and it is also the measure of normal direction n. For the purpose of computation φ is designated as φ_j^i, where $i\,(=1,\ldots,N_L)$ is the measure of load and $j\,(=1,\ldots,N_g/N_f)$ corresponds to the location, where φ is measured in x/s coordinate system. When large deflection analysis is carried out in Cartesian coordinate system (x, y), the curvature is given by $\frac{1}{\rho} = \frac{d^2y}{dx^2}\Big/\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\left(\frac{3}{2}\right)}$. However in the analysis of small deflection problems, the curvature is approximated as $\frac{1}{\rho} = \frac{d^2y}{dx^2}$, and as a consequence the domain of x becomes $0 \le x \le L$, i.e., the beam stretches with increase in loading as shown in Fig. 1(a). On the other hand, in large deflection bending analysis of cantilever beams, it is assumed that the length of the beam does not change with loading. Hence the domain of s remains unchanged and spans from 0 to L ($0 \le s \le L$). To maintain constancy in beam length, the domain of x changes with loading, spanning from 0 to the projected length l of the beam, as shown in Fig. 1(b). The first derivative of equation (1) with respect to s, yields,

$$EI\frac{d^2\varphi}{ds^2} = \frac{dM}{ds} \tag{2}$$

The bending moment M at location s is,

$$M(s) = F(l - x). \tag{3}$$

Differentiating equation (3) with respect to s, and comparing with equation (2) the following non-linear differential equation is obtained, taking into account the geometrical relations $\cos\varphi = \frac{dx}{ds}$ and $\sin\varphi = \frac{dy}{ds}$.

$$EI\frac{d^2\varphi}{ds^2} + F\cos\varphi = 0 \tag{4}$$

Equation (4) is multiplied by $\frac{d\varphi}{ds}$ to yield $EI\frac{d\varphi}{ds}\frac{d^2\varphi}{ds^2} + F\cos\varphi\frac{d\varphi}{ds} = 0$ and after carrying out some mathematical manipulations, it is expressed as

$$\frac{d}{ds}\left[\frac{EI}{2}\left(\frac{d\varphi}{ds}\right)^2 + F\sin\varphi\right] = 0 \tag{5}$$

Equation (5) is integrated and the associated constant of integration is evaluated by using boundary conditions (i) $\varphi = \varphi_{tip}^{N_L}$ and (ii) $\frac{d\varphi}{ds} = 0$ at $s = L$. $\varphi_{tip}^{N_L}$ represents the slope $\frac{dy}{dx}$ corresponding to load F at load step number N_L. Hence equation (5) becomes

$$\left(\frac{d\varphi}{ds}\right)^2 = \frac{2F}{EI}\left(\sin\varphi_{tip}^{N_L} - \sin\varphi\right) \tag{6}$$

Using a normalized load parameter $\alpha\left(=\frac{FL^2}{2EI}\right)$, the above equation is expressed as

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