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Computational solutions for non-isothermal, nonlinear magneto-convection in porous media with hall/ionslip currents and ohmic dissipation

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ABSTRACT

A theoretical and numerical study is presented to analyze the nonlinear, non-isothermal, magnetohydrodynamic (MHD) free convection boundary layer flow and heat transfer in a non-Darcian, isotropic, homogenous porous medium, in the presence of Hall currents, Ionslip currents, viscous heating and Joule heating. A power-law variation is used for the temperature at the wall. The governing nonlinear coupled partial differential equations for momentum conservation in the x and z directions and heat conservation, in the flow regime are transformed from an (x, y, z) coordinate system to a (ξ,η) coordinate system in terms of dimensionless x-direction velocity $(\partial F/\partial \eta)$ and z-direction velocity (G) and dimensionless temperature function (H) under appropriate boundary conditions. Both Darcian and Forchheimer porous impedances are incorporated in both momentum equations. Computations are also provided for the variation of the x and z direction shear stress components and also local Nusselt number. Excellent correlation is achieved with a Nakamura tridiagonal finite difference scheme (**NTM**). The model finds applications in magnetic materials processing, MHD power generators and purification of crude oils.

1. Introduction

Heat transfer in the presence of strong magnetic fields is important in various branches of magnetohydrodynamic power generation [1], nanotechnological processing [2], nuclear energy systems exploiting liquid metals [3] and blood flow control [4]. Hall currents and Ionslip effects become significant in strong magnetic fields and can considerably affect the current density in hydromagnetic heat transfer. Joule heating effects are also important and are caused by heating of the electrically-conducting fluid by the electrical current. As such considerable attention has been devoted to studying hydromagnetic convection flows with such effects. Mazumder [5] presented exact solutions for Hall current effects in rotational hydromagnetic flow due to the non-torsional oscillation of a porous plate. He investigated in detail both the steady and transient velocity fields and multiple boundary layers. Rao and Mittal [6] studied the incompressible hydromagnetic boundary layer in an MHD generator configuration using a Runge Kutta method. Hossain [7] reported on the effects of Hall currents on transient natural convection MHD boundary layer with suction at the wall using the Keller-Box numerical method. Raju and Rao [8] studied the cases of conducting and non-conducting walls for ionized hydromagnetic rotating heat transfer in a parallel plate channel with Hall currents. They showed that the temperature field is independent of partial pressure of electron gas for the case of non-conducting walls. Increasing values of rotation parameters were found to reduce the temperatures in the channel for constant Hartmann number and Hall current parameter. Sawaya et al. [9] determined experimentally the Hall parameter for electrolytic solutions in a closed loop thermosymphonic magnetohydrodynamic flow system. A one dimensional theoretical model with the measured open circuit voltage was used to quantify Hall parameter. Bhargava and Takhar [10] studied computationally the influence of Hall currents on hydromagnetic heat transfer of a viscoelastic fluid in a channel. These studies did not consider however the presence of ion-slip currents. In weaker magnetic fields, the diffusion velocity of electrons and ions is different and usually ionslip effects are neglected. However in MHD generators and industrial materials processing where the electromagnetic body forces are large (i.e. strong magnetic fields present), the diffusion velocity of the ions cannot be

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neglected. When both electron and ion velocities are incorporated in the analysis, the ionslip phenomenon is present and Ohm's law has to be modified accordingly. An excellent discussion of ionslip effects has been presented by Cramer and Pai [11]. Soundalgekar et al. [12] were among the first researchers to consider ionslip effects in hydromagnetic Couette heat transfer in addition to Hall currents effects. They showed that for small magnetic field parameters or high ionslip and Hall current parameters, the flow can become unstable. A reverse in flow was observed with strong Hall and ionslip effects. Strong ionslip was shown to increase temperatures whereas a rise in Hall parameter was shown to reduce temperatures. Nusselt number was shown to increase considerably with a rise in magnetic parameter but only initially to increase then decrease with ionslip parameter. Ram and Takhar [13] reported on the rotating natural convection MHD flow with Hall/ionslip current effects. Ram et al. [14] extended this study to consider the effects of oscillating wall temperature using a numerical method. Further studies of combined Hall and ionslip currents in magnetohydrodynamic heat transfer flows were provided by Takhar and Jha [15] and more recently by Elshehawey et al. [16]. M. Turkyilmazoglu [17] examined the exact solutions for the incompressible viscous MHD fluid of porous rotating disk flow with Hall current. M. Turkyilmazoglu [18] studied the exact solutions for the incompressible viscous MHD fluid of rotating disk flow with Hall current. Several studies of Joule electrical heating in MHD heat transfer flows have also appeared. Michiyoshi and Matsumoto [19] presented one of the first studies of hydromagnetic heat transfer with Joule heating. They studied the Joule heating effects on laminar parallel plate channel hydromagnetic heat transfer in the thermal entrance region. Both prescribed uniform wall heat flux and uniform wall temperature cases were considered. Wu and Cheng [20] used an eigenfunction expansion method to investigate the combined effects of Joule heating and axial conduction on thermal entry Hartmann heat transfer and flow in a parallel plate channel with different wall temperatures. They studied the case of an open circuit and considered Hartmann number up to 10 and Brinkmann numbers of 0 and −1. Mansour and Gorla [21] more recently studied the effects of Joule heating effects on transient free hydromagnetic convection in a micropolar fluid. Bég [22] studied the effects of Joule heating in MHD channel flow using a Navier–Stokes computational solver. Aissa and Mohammadein [23] more recently analyzed the effects of Joule heating and variable electric conductivity on micropolar stretching flow and heat transfer using a shooting numerical scheme. The combined effects of Hall current, magnetic induction and oblique magnetic field on MHD flow in a spinning channel with heat transfer were studied by Ghosh et al. [24]. Other studies incorporating Joule heating have been communicated by Duwairi [25] and Zueco et al. [26] who employed an electrothermal network simulation code. M. Rahimi-Gorji et al. [27] analyzed the unsteady motion of vertically falling spherical particles in non-Newtonian fluid by collocation method. Simulation of magnetic drug targeting through tracheobronchial airway in presence of an external nonuniform magnetic field using Lagrangian magnetic particle tracking was studied by O. Pourmehran et al. [28]. O. Pourmehran et al. [29] investigated the squeezing unsteady nanofluid flow between parallel plates by LSM and CM. O. Pourmehran et al. [30] studied the optimization of microchannel heat sink geometry cooled by different nanofluids using RSM analysis.

In many industrial and geophysical flows viscous dissipation effects may also arise owing to internal friction in viscous fluids which can affect temperature fields. Many studies have been reported concerning viscous heating effects in both natural and forced convection heat transfer flows. These include the articles by Gebhart and Mollendorf [31] and Soundalgekar and Pop [32] which dwell on boundary layer heat transfer. In hydromagnetic heat transfer several studies have been reported concerning viscous heating effects. Javeri [33] studied hydromagnetic heat transfer in a channel with the collective effects of Hall current, ion slip, viscous dissipation and Joule heating. Takhar and Soundalgekar [34] presented numerical solutions for the effects of Eckert number (viscous heating parameter) on hydromagnetic natural convection boundary layer flow. Other non-magnetic studies of viscous heating effects include those by Turcotte et al. [35], Basu and Roy [36], Barletta [37] and Barletta and Rossi di Schio [38]. These studies have all been restricted to purely fluid regimes. In numerous systems the medium may be a porous material. The porosity of materials is an intrinsic aspect of many chemical engineering and materials processing systems. Ceramics, batch reactors, purification systems and filtration systems all utilize porosity. Traditionally the Darcian model has been employed to analyze most convection flows in porous media. Such a model however is generally only accurate at very low Reynolds numbers and cannot simulate the inertial effects experienced at higher Reynolds numbers. Engineers have therefore extended the Darcian model to incorporate second order drag force effects generally with the Forchheimer-extended Darcian model, which is easily implemented in boundary layer heat transfer analysis. Excellent studies of Darcy–Forchheimer convection in porous media have been presented by for example, Chen and Ho [39] and also Manole and Lage [40]. O. Pourmehran et al. [41] examined the optimization of microchannel heat sink performance cooled by KKL based in saturated porous medium.

The above studies however did not consider the *collective effects* of Joule heating, Hall or ionslip currents in porous media transport phenomena. The vast majority of simulations employ a *Darcian* model [42], valid for viscous-dominated flows. In this paper therefore we shall consider the *composite effects of Joule heating, Hall and ionslip currents, and also viscous frictional heating on two-dimensional natural MHD convection in a Darcy–Forchheimer porous medium from a vertical plate with power-law variation in the wall-temperature*. Such a study has thus far not appeared in the literature and constitutes a useful extension to the current body of work on non-linear magneto-convective transport phenomena in porous media.

2. Mathematical model

We consider the steady state hydromagnetic natural convection flow of a viscous, incompressible, partially-ionized, electrically-conducting fluid flowing adjacent to a non-isothermal vertical surface in an (x, y, z) coordinate system embedded in a non-Darcy saturated porous medium. The plate surface is in the x-z plane. The z-axis coincides with the leading edge of the plate. A strong magnetic field acts parallel to the y-axis. The physical model is illustrated in Fig. 1. The magnetic Reynolds number is small for the partially-ionized fluid so that magnetic induction effects can be ignored. However, relative motion of the particles in the fluid can occur and the electron–atom collision frequency is assumed to be high enough for Hall and ionslip currents to be significant. As such, an electric current density, $\boldsymbol{J}$, is required to represent the relative motion of charged particles. Considering only the electromagnetic forces on these particles, we can utilize the generalized Ohm law. With a magnetic field, $\boldsymbol{B}$, applied normal to the electrical field $\boldsymbol{E}$, an electromagnetic force is generated normal to both $\boldsymbol{E}$ and $\boldsymbol{B}$ in the z direction. Such a force causes charged particles to migrate perpendicularly to both $\boldsymbol{E}$ and $\boldsymbol{B}$ [11]. Consequently, a component of electrical current density exists perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$, and this constitutes the Hall current. For a strong magnetic field $\mathbf{B}$ the diffusion velocity of the ions will be significant, constituting the ionslip effect. From the equation of conservation of electrical charge:

$$\nabla \cdot \mathbf{J} = 0 \tag{1}$$

where $\boldsymbol{J} = (J_x, J_y, J_z)$. Since the plate is not composed of electrically-conducting material, electrical charge at the surface of the plate is