European Journal of Operational Research 255 (2016) 364–379

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

# Optimal production-inventory policy for an integrated multi-stage supply chain with time-varying demand

Shi Tao Zhao[a,*], Kan Wu[a], Xue-Ming Yuan[b]

[a] School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore
[b] Singapore Institute of Manufacturing Technology, Singapore

ARTICLE INFO

*Article history:*
Received 29 September 2014
Accepted 14 April 2016
Available online 21 April 2016

*Keywords:*
Supply chain management
Inventory
Time-varying demand
Networks

ABSTRACT

An integrated multi-stage supply chain with time-varying demand over a finite planning horizon is considered in this paper. The objective is to devise the optimal production-inventory policy to minimize the total operational cost. The model is formulated as a mixed integer nonlinear programming problem. The problem is represented as a weighted directed acyclic graph. The global minimum total operational cost is computed in polynomial time by the developed algorithm. Two numerical examples of a seasonal product and a product over its life cycle are studied to illustrate the results. A sensitivity analysis of the system parameters is conducted to assist the supply chain decision makers.



## 1. Introduction

As per the definition by The Council of Supply Chain Management Professionals, supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and logistics management activities. To accomplish these activities, conflicting issues arise when supply chain parties make decisions in isolation. Current supply chains are facing complexity from the multi-stage hierarchy and uncertainties in supply and demand. To meet these challenges, supply chain parties must work toward an integrated system and coordinate with each other.

Integrating various processes in a supply chain to devise the optimal production-inventory policy is essential in a competitive industrial environment. Since the 1990s, numerous studies have been conducted on integrated supply chains to minimize the total operational costs by finding the optimal lot sizes (Ben-Daya, Darwish, & Ertogral, 2008; Brahimi, Dauzere-Peres, Najid, & Nordli, 2006; Glock, 2012). However, most of them considered the constant demand. They cannot reflect a practical situation because the demand rates are changing with time due to seasonal variation, business cycle, irregular fluctuation, and product life cycle. Seasonal variations result from both natural randomness and human decisions. For example, umbrellas are in high demand during the monsoon period in Southeast Asia. Customers may change their behaviors during an umbrella sales promotion. So, the demand for umbrellas is subject to seasonal variations. In a typical business cycle, the economy slowly moves through stages of expansion, slowdown, recession, and recovery. The demand for most products is positively correlated with the business cycle. Unpredictable events such as accidents and natural disasters may lead to irregular fluctuations in demand. The product life cycle can be divided into four discrete stages: introduction, growth, maturity and decline (Aitken, Childerhouse, & Towill, 2003). The product demand is dynamically correlated with its life cycle, particularly for high-tech industries (Marquez & Blanchar, 2006; Teng, Min, & Pan, 2012). For time-varying demand, linearization techniques can be applied to approximate the actual demand curve over the planning horizon, which consists of multiple phases. The approximation is accurate if the interval is sufficiently small (Andriolo, Battini, Grubbstrom, Persona, & Sgarbossa, 2014; Hill, 1995; Silver, 1979; Silver, Pyke, & Peterson, 1998). Therefore, the multi-phase function shown in Fig. 1 can represent the demand for most products. It is composed of piece-wise linear demand patterns over a planning horizon $T_f$, where $f$ is the index of phases, $f = 1, 2, 3, \ldots, F$.

To adapt to demand fluctuations, supply chain decision makers should carefully manage their inventories and constantly monitor the production schedule so that the total operational cost can be minimized. Indeed, effectively managing the supply chain with time-varying demand is an important issue for current multinational corporations (MNCs).

In this paper, we develop a new model for an integrated multi-stage supply chain with time-varying demand over a finite planning horizon. The supply chain includes supply, production,

* Corresponding author. Tel.: +65 6790 5933/65 9879 0895; fax: +65 6790 1619.
*E-mail addresses:* zhao0119@e.ntu.edu.sg (S.T. Zhao), Kan626@gmail.com (K. Wu), xmyuan@SIMTech.a-star.edu.sg (X.-M. Yuan).

http://dx.doi.org/10.1016/j.ejor.2016.04.027
0377-2217/

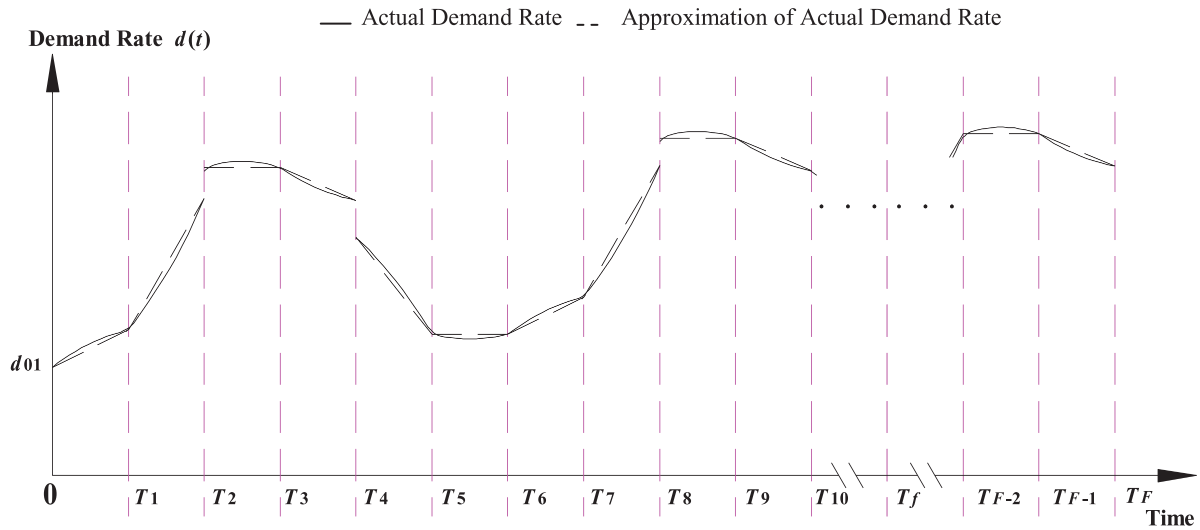


**Fig. 1.** The timer-varying multi-phase demand pattern.

transportation and distribution. The objective is to devise an optimal production-inventory policy to minimize the total operational cost which includes production setup cost, product ordering cost, inventory holding cost, and transportation cost. We synchronize the inventories at different stages of the integrated multi-stage supply chain before it is formulated as a mixed integer nonlinear programming (MINLP) problem. The problem is represented as a weighted directed acyclic graph (WDAG). The global minimum total operational cost is computed in polynomial time. A conventional assumption for many models in previous studies is that the holding cost rates increase as the material/product flows down the supply chain(Glock, 2012; Kaminsky & Simchi-levi, 2003; Kim & Glock, 2013; Lee, 2005; Liu & Lian, 2009; Zhao, Wu, & Yuan, 2016) because the product increases in value as it moves down the supply chain. However, some studies show that this is not true in several industries, such as the aircraft, automotive, personal computer and retailer supply chains with consignment stock policies (Braglia & Zavanella, 2003; Chen, Lin, & Cheng, 2010; Diabat, 2014; Valentini & Zavanella, 2003; Verma, Chakraborty, & Chatterjee, 2014; Yi & Sarker, 2014). Our model is more generic than the existing models in previous studies as this assumption is relaxed in this paper.

The rest of this paper is structured as follows. The next section reviews the literature that is closely related to our research. Section 3 describes the problem. The total operational cost function is formulated in Section 4. In Section 5, the model is represented as the WDAG. The global minimum total operational cost is computed by the developed algorithm. In addition, some special cases are discussed in this section. Two industrial applications of the time-varying demand, a seasonal product and a product over its life cycle are illustrated in Section 6. Section 7 presents the paper's conclusions.

## 2. Literature review

Based on the nature of demand, production-inventory problems can be classified as deterministic or stochastic (Axsäter, 2006; Cachon & Fisher, 2000; Cachon & Lariviere, 2005; Chen, Pekny, & Reklaitis, 2013; Liu & Lian, 2009; Sana & Goyal, 2014; Song, Dong, & Xu, 2014; Zhao, Xu, Zhang, & He, 2007; Zipkin, 2000). In the deterministic class, it can be further classified into constant demand and time-varying demand. There have been several studies on time-varying demand with the assumption of an infinite production rate or an instantaneous inventory replenishment for integrated supply chains (Hwang, Ahn, & Kaminsky, 2013; Kaminsky & Simchi-levi, 2003; Kian, Gurler, & Berk, 2014; Onal, Romeijn, Sapra, & van den Heuvel, 2015; Van Hoesel, Romeijn, Morales, & Wagelmans, 2005; Ventura, Valdebenito, & Golany, 2013; Zangwill, 1969) since the classical article by Wagner and Whitin (1958). However, the optimal production-inventory policy for the supply chain with the infinite production rate or the instantaneous inventory replenishment might not be feasible or lead to excessive work-in-process in the actual manufacturing environment (Almeder, Klabjan, Traxler, & Almada-Lobo, 2015). The recent studies (Almeder et al., 2015; Buschkuehl, Sahling, Helber, & Tempelmeier, 2010; Clark, Almada-Lobo, & Almeder, 2011) pointed out that it is more realistic to consider the positive lead time in the multi-stage supply chain.

Inspired by the practice in the production-inventory problems, many researchers focused on supply chains with a linear trend in demand and a finite production rate. For instance, Hariga (1996) studied an optimal inventory model of deteriorating items with a linear trend in demand. Lo, Tsai, and Li (2002) presented the exact optimal solution of inventory replenishment for both linear increasing and decreasing trends in demand. Rau and OuYang (2008) presented an optimal production–inventory policy for a two-stage supply chain with linear increasing and decreasing trends in demand. Although the single-phase linear trend in demand is an approximation of the practical situation (Chang & Chou, 2013; Chen & Chang, 2007), the demand for most products has a typical cycle with the multi-phase patterns due to the life cycle or boom-and-bust seasons in a year. Hill (1995) considered a product subject to a period of increasing demand, followed by a period of constant demand for a single-stage system. Following the same demand pattern as Hill (1995), Teng et al. (2012) developed an economic order quantity model with trade credit financing with increasing demand for a single-stage system. Diponegoro and Sarker (2007) studied a production-supply problem for a two-stage supply chain with a fixed interval delivery to buyers. The product demand is time-dependent following its life cycle. However, the multi-phase demand over the planning horizon is represented by a piece-wise linear function up to three phases. Sarker and Balan (1999) studied a multi-stage kanban system with a linear trend in demand. Goyal and Giri (2003) considered a production-inventory model with time-varying demand to devise the optimal replenishment policies. Sana (2010) also developed a production-inventory model to determine the optimal product reliability and optimal production rate that achieves the