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An EOQ model for perishable products with fixed shelf life under stochastic demand conditions

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ABSTRACT

The paper presents a mathematical stochastic model for perishable open-dating foods including shortage and outdating costs. The demand fluctuations have been taken into account modeling them through a normal distribution, and their impact on the storage time has been studied considering the stochastic nature of such a parameter in turn. The quantification of perished products has been also addressed, determining the probability for a product of remaining in stock beyond the end of its Shelf Life. On the basis of such premises, the optimal set of parameters that minimize the total unit cost has been determined. A numerical application and a sensitivity analysis show the practical applicability of the proposed model in the context of warehouse management, underlining managerial insights.

1. Introduction and literature review

Inventory models for deteriorating items have attracted considerable interest in recent decades. The problem of modeling the deterioration process was firstly tackled by Ghare and Schrader (1963) that developed an exponentially decaying inventory model. They observed that certain commodities deteriorate with time by a proportion which can be approximated by a negative exponential function of time. Successively, Covert and Philip (1973), considered a two parameters Weibull deterioration function. Since the work of Ghare and Schrader (1963) and Covert and Philip (1973), considerable works have be done on deteriorating inventory systems, which are summarized in Raafat (1991), that dealt with 70s and 80s in relation to continuously deteriorating items. Goyal and Giri (2001), extended the review at 90s. Nahmias (2011), presented a review referred to fixed and random lifetime models. Finally, Karaesmen, Scheller-Wolf, Deniz, and Civelek (2008), Li, Lan, and Mawhinney (2010), and Bakker et al. (2012), considered the inventory theory with regards to the latest results in such field. In particular Bakker et al. (2012), proposed a review that extended the literature to the work done in the field of perishable inventory, since the 2001 until the 2012. Such literature considered the two fundamental decisions that affect the inventory related costs, in managing perishables: replenishment and issuance.

The question generally dealt in perishable inventory research field is to determine the optimal batch to stock under either deterministic or stochastic conditions of the demand and possibly considering constant or time-dependent deterioration (e.g., exponential, Weibull or Gamma deterioration distribution) and shortage costs (see for example Begum, Sahoo, & Sahu, 2010; Chang et al., 2010; Ghosh & Chaudhuri, 2005; Khanra et al., 2010; Mahata and Goswami, 2009; Manna & Chaudhuri, 2006; Mishra & Shah, 2008; Mishra & Singh, 2011; Ou & Min, 2010; Roy & Chaudhuri, 2011; Roy et al., 2011; Roychowdhury, 2009; Sana 2011, for deterministic models and Aggoun, Benkherouf, & Tadj, 2001, Brander & Forsberg, 2006; Broekmeulen van Donselaar, 2007; Hala & El-Saadani, 2006; Halim, Giri, & S., 2008; Hayya, Harrison, & Chatfield, 2009; Søren & Hill, 2000, for stochastic ones). The literature mentioned is particularly suited for fruits and vegetables. In this case, deteriorated units can be identified through manual inspections. However, today emerging technologies such as the Radio Frequency Identification (RFID) with embedded sensors allow us to perform automatic inspections through the application of mathematical models relying on time-temperature history of products. On the contrary, such an approach is not well suited for “open dating” or canned foods, like those managed at the retailing stage of the food chain (typically iper and super markets), governed by the “use by date” or “sell by date”. Such a date indicates that the products remain safe and suitable for human consumption until the reaching of the end of their Shelf Life (SL). Once this time is achieved, they are to be considered completely perished and consequently discarded. For that a reason, such products are considered having a deterministic and fixed life. In this paper, the SL is addressed as the remaining SL of products as they enter the retailing stage, namely a portion of the total lifetime starting form the end of the process/manufacturing cycle, as better explained in Kilcast and Subramaniam (2000).

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Works found in literature do not consider the link between the deterioration rate of products and their SL and mark as outdated those products still in stock at the end of the cycle time (defined as the time between consecutive batches, see Diwekar, 2014), without verifying that the cycle time is greater or less with respect to the SL of products. In practice, such an essential relationship is usually guaranteed avoiding of managing products with very short SL. Thus, the outdating units are calculated as the amount of partially deteriorated products remaining in stock at the end of the cycle time. Such products are usually discarded at the end of the cycle time, or someway recovered and sent to markets with less stringent quality standards, at discounted prices. However, being the cycle time less than the SL, such products should not be considered outdated, but still suited for the target market. In such context, the approaches proposed in literature until now are not compliant with supply chain goals, that advocate cost efficiency as a requirement to satisfy. Moreover, while such approaches have a low impact on the fresh products such as fruits and vegetables, whose SL is uncertain and, as a result, it can be determined only by manual or automatic inspections, it has instead a strong impact on packaged products, with the result of loss of profit for the firm carrying out such a policy.

Conversely, when the assumption of the overcoming of the SL during the storage time is allowed (i.e., the cycle time is greater than the SL), products in stock are to be considered deteriorated and necessarily discarded.

The literature analysis allowed us to find only few papers in which the SL of products is taken into consideration in the optimization function. This is the case of Avinadav and Arponen (2009), that discussed an extension of the classical Economic Order Quantity (EOQ) model for items with a fixed SL and a deterministic declining demand rate due to a reduction in the quality of the item in the course of its SL, and Yan (2012), that presented an EOQ model for perishable items with freshness-dependent demand. However, they neglect of considering the outdating units in stock as the consequence of the case in which the cycle time is greater than the SL. Therefore, the main shortcoming found in the review of literature relates to the possibility that the SL of products is overcome during the cycle time and, in such context, the main contribution of the present paper is to propose an EOQ model for perishable products, considering the SL as a distinctive parameter to determine the amount of perished products still in stock. The goal is to provide a mathematic model for supply chains managing "open-dating" foods, which allows us to remove outdating costs related to the discard of still suitable products. The perishability of products is addressed in terms of the probability that a product remains in stock beyond the achievement of its SL.

Moreover, the literature analysis led to find EOQ models taking into consideration freshness-dependent demand (see for example Avinadav & Arponen, 2009; Avinadav, Herbona, & Spiegel, 2013; Bai & Kendall, 2008; Piramuthu & Zhou, 2013; Yan, 2012). In the present paper the demand variability resulting from consumer preferences, based on weather conditions, freshness of products, micro–macro economic factors etc., is taken into consideration by enforcing the assumption of stochastic demand behaviour. The impact of the stochastic demand behavior is determined not only in terms of stockout costs and safety stock, but also considering the storage time as a stochastic variable consequently. The stock on hand satisfying a desire Cycle Service Level (CSL) (i.e. the probability from available stock during the replenishment cycle (Li, 2007)) is fixed and the safety stock is determined, according to the demand variability. Given the accepted risk (1-CSL) (i.e. the probability of having a stockout during the replenishment cycle) that the market demand overcomes the stock on hand during the cycle time, the stockout costs are calculated. Information about the SL of products is employed to calculate the amount of shortages. Accordingly, the proposed model determines the optimal cycle time and the quantity to be stocked on hand including the shortage and deterioration costs.

2. Model formulation

The model here presented deals with products having a deterministic and constant SL. In the case of deterministic demand behaviour such an assumption would lead to the purchasing of the quantity sold within the time of the SL, i.e., $EOQ = demand\ rate * SL$ and $t_1 = T_c = SL$. Conversely, the stochastic behavior of the demand here enforced leads two main results. First of all, it determines the possibility of stockout occasions due to the demand variability. Such problem can be partially overcome by stocking an additional quantity of products, namely the safety stock, for a desired CSL. Moreover, the demand variability leads that the time the products are held in stock can be considered in all respects a random variable as well, whose expected value is the SL of the product. The second problem relates with the possibility of quantifying the perished units in stock. Unlike the models usually present in literature, relying on the well known differential equation including the deterioration rate $\theta(t)$ (see for example Mishra & Shah, 2008), we propose of determining the amount of perished products as the number of products remaining on hand beyond their SL, i.e., calculating the probability that the products remain in stock beyond the end of their SL. The inventory model relates to the determination of optimal parameters consisting in the optimal batch size, the time the products are held in stock and cycle time, allowing at minimizing the total unit management cost of products stocked.

2.1. Assumptions and notation

The model has been formulated by considering the following assumptions and notation:

- The inventory system deals with one item.
- The item has a shelf life equal to *SL*, that is deterministic and constant, meaning that one item can be sold until the ending of the *SL*, when it must be considered completely perished and discarded.
- Shortages are allowed and the demand completely lost.
- t_1 is the time at which the inventory level drops to zero, a random variable that must be optimized.
- The lead time (*LT*), i.e., the amount of time between the placement of the order and its receipt is deterministic and constant. Stated this assumption, the *LT* cannot impact the optimality conditions of *SL* of products handled at the retailing stage, as in fact if the *LT* is such that the remaining *SL* of products is less that that established, the products are rejected and sent back to the wholesaler. This is consistent with the contractual policies between retailers and wholesalers, in which the latter actor have to guarantee a specified remaining *SL* to the first one at the time at which the products enter the retailing stage (see Garrone et al., 2012). Nevertheless, the *LT* will be considered in the model presented.
- The expected demand rate is assumed to be a random variable normally distributed with mean μ_t and variance σ_t^2, referred to the time unit *t*. During the *LT*, the demand is equal to $\mu_t LT$.
- The safety stock is considered in order to ensure a fixed CSL and it is equal to $F^{-1}(CSL)\sigma_t$. In the case of normally distributed demand, such a quantity can be approximated by $k\sigma_t$, where *k* is the safety factor (see Alstrøm, 2001; Annadurai & Uthayakumar, 2010; Brander & Forsberg, 2006; Chung, Ting, & Hou, 2009). During the *LT*, the safety stock is equal to $k\sigma_t LT$.
- *A* is the setup cost per order.
- *h* is the inventory holding cost per unit held in stock per unit time.