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Optimal production planning for assembly systems with uncertain capacities and random demand

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A B S T R A C T

We study the optimal production planning for an assembly system consisting of n components in a single period setting. Demand for the end-product is random and production and assembly capacities are uncertain due to unexpected breakdowns, repairs and reworks, etc. The cost-minimizing firm (she) plans components production before the production capacities are realized, and after the outputs of components are observed, she decides the assembly amount before the demand realization. We start with a simplified system of selling two complementary products without an assembly stage and find that the firm's best choices can only be: (a) producing no products or producing only the product of less stock such that its target amount is not higher than the other product's initial stock level, or (b) producing both products such that their target amounts are equal. Leveraging on these findings, the two-dimensional optimization problem is reduced to two single-dimensional sub-problems and the optimal solution is characterized. For a general assembly system with n components, we show that if initially the firm has more end-products than a certain level, she will neither produce any component nor assemble end-product; if she does not have that many end-products but does have enough mated components, she will produce nothing and assemble up to that level; otherwise she will try to assemble all mated components and plan production of components accordingly. We characterize the structure of optimal solutions and find the solutions analytically.

1. Introduction

In manufacturing, capacity can be uncertain due to factors such as unexpected breakdowns of unreliable machinery, unplanned maintenance of uncertain duration, rework of randomly defective items etc. (Ciarallo, Akella & Morton 1994, Hu, Duenyas & Kapuscinski 2008). Such uncertainties complicate production planning in that the output is not necessarily equal to the planned amount. The effect of uncertain capacities on production planning in serial systems has been investigated (see e.g., Hwang & Singh 1998, Wang & Gerchak 1996a); however, little is known for assembly systems which happen to be more prevalent in modern manufacturing.

We study optimal production planning in a single period setting for an assembly system where demand for the end-product is random and all components production and end-product assembly capacities are uncertain. Given abundant input, each item's (components and the end-product) output is random and equal to the minimum of its planned amount and the realization of its uncertain capacity. The firm chooses production amounts of components before production capacities are realized, and after outputs of components are observed, she decides the planned assembly amount before realization of the assembly capacity and demand. Unmet demand incurs penalty cost while unused items result in disposal costs. Production and assembly costs are charged based on outputs of components and the end-product, respectively.

A direct motivation of our paper comes from a problem faced by cell phone producers in China. In the highly-competitive cell phone market, all competitors strive to introduce new products quite frequently to stimulate demand and attract consumers. A producer's ability of having abundant new products ready for a coming selling season is a precondition for the success of her business. One of cell phone producers' difficulties is that their components production suffers from uncertain capacities due to increased internal uncertainties arising from immature technologies, complex control and lack of production experience, etc. As a result, late deliveries of components may happen and firms lose not only sales but also valuable market shares. Nonetheless, firms are reluctant to use expensive mitigation measures such as stockpiling and backup capacity/supplier because margin is dropping throughout the

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industry. They need to plan their productions by taking into account not only random demand but also uncertain capacities. One may find cell phone producers' story echoes in other competitive industries (e.g., innovative consumer electronics) where the end-products have short life-cycles. Our model presented here tries to address the production planning problems in such environments.

We start with a system where the firm sells two perfectly complementary products as a set to the market. This corresponds to a two-component system where assembly is unnecessary. We show that the optimal production plan can only be: (a) producing no products at all or producing only the product of less stock such that its target amount is at most as high as the other product's initial stock level, or (b) producing both products such that their target amounts are equal. We then use these facts to reduce the two-dimensional optimization problem into two interconnected one-dimensional sub-problems that are analytically tractable. Each sub-problem is solved and the optimal solution to the original problem is then fully characterized. We then extend the model to consider a general assembly system with n components where the firm engages in two-stage decision-making (components production planning in stage 1 and end-product assembly planning in stage 2) with uncertain production and assembly capacities. We find that if the initial stock of end-product is lower than a certain level, she will act in an assemble-as-many-as-possible fashion in stage 2 and will choose components' planned amounts accordingly in stage 1. Otherwise, she will neither assemble any end-product nor produce any components. When producing is necessary for some or all components, we fully characterize the optimal solution.

This research is closely related to the literature on production/inventory systems under uncertain capacities. In the seminal work of Ciarallo et al. (1994), they show that for a single-stage, single-item system with uncertain capacity and random demand, an order-up-to policy is optimal for a periodic review finite-horizon problem, and a so-called "extended myopic" policy(which requires the consideration of review periods of uncertain length) is optimal for the infinite horizon problem. For the same system, Güllü (1998) presents a procedure for computing the optimal base stock level under expected average cost per period criterion, while Iida (2002) extends the model to consider non-stationary demand and develops upper and lower bounds of optimal policies for infinite horizon problems. Wang and Gerchak (1996a) consider uncertain capacity as well as random yield and show that the optimal policy is of a reorder-point type. Erdem and Özekici (2002) develop a rather general periodic-review model that incorporates both uncertain capacity and random environment which is a time-homogeneous Markov chain. They show that the optimal policy is the well-known base-stock policy where the optimal order-up-to level depends on the state of the environment. For a single-period multi-stage system with setup costs and uncertain capacities at all stages, Hwang and Singh (1998) prove that the optimal policy is characterized by a sequence of imbedded critical numbers. Hu et al. (2008) address the optimal joint control of inventory and transshipment for a firm that produces in two locations whose capacities are uncertain. Under Markovian production capacity, Yang, Qi and Xia (2005) study the optimal inventory-outsourcing policy while Yang, Qi and Xia (2006) study inventory control with an option of order rejection. Under the EOQ framework, Wang and Gerchak (1996b) analyze the effects of uncertain capacity on optimal lot sizing with single supplier while Erdem, Fadiloglu and Özekici (2006) consider multiple suppliers. From a strategic point of view, in a single period setting, Wang, Gilland and Tomlin (2010) examine which mitigation strategy, dual sourcing or process improvement, is favorable for a firm facing uncertain capacity or random yield as supplier cost/reliability varies.

The above-mentioned papers are mainly about serial systems, little is known about the optimal policies for assembly systems with uncertain capacities. Bollapragada, Rao and Zhang (2004) study an assembly system with uncertain capacities at component stage, but their system is assumed to be operating under a periodic-review installation base-stock policy, and their goal is to minimize the total steady-state surrogate holding cost subject to a service level constraint. We, on the other hand, aim to fully characterize the optimal solution for the system. Xiao, Chen and Lee (2010) is another paper on assembly systems with uncertain capacity, but the uncertainty lies in only the assembly stage. In our system, both the component production capacities and assembly capacity can be uncertain. Recently, Bollapragada, Kuppusamy and Rao (2015) numerically illustrate the impacts of lead times and capacity on the performance of a complicated assembly system with supply uncertainty. However, no optimal policies or behaviors are provided.

In a broader sense, our paper is also related to research on production/inventory decisions in assembly systems with random yield (another common internal cause for random output besides uncertain capacity). In his seminal work, Yao (1988) applies a chance-constrained optimization approach to minimize the cost of procuring components subject to a given probability of achieving a fixed target yield on the assembly stage. Gurnani, Akella and Lehoczky (2000) extend Yao's model to optimize components ordering and assembly target level decisions simultaneously under random demand. Gerchak, Wang and Yano (1994) address lot sizing decisions in assembly systems with random yield both at component stage and assembly stage, they prove that the cost functions are jointly concave under certain circumstances and the optimal lot sizes are then given by the first order conditions. Pan and So (2010) study joint pricing-production decisions in a two-component assembly system where one of the components faces random yield. All of these papers are conducted in a single-period setting except for DeCroix (2013), in which the optimal policies for an assembly system subject to supply disruptions (considered as a special "all-or-nothing" case of random yield) are characterized.

In these random yield models, the optimal solutions are about "inflating the planned production amounts". Moreover, the targeted amounts (equal to planned amount plus initial inventory) of the complementary components are generally not equal. These are not true in our uncertain capacity model. We found that in assembly systems with uncertain capacities, the optimal solutions are about "deflation" rather than "inflation" of the planned production quality. That is, the planned amounts under uncertain capacities are generally smaller than those under unlimited capacities. Moreover, a wise decision-maker will plan production such that the targeted amounts of different components are equal. The key to such differences is that capacity uncertainty implies that output is the minimum of the planned amount and the realized capacity while yield randomness often implies that output is stochastically proportionate to the planned amount.

The paper is organized as follows. For ease of exposition, we start in Section 2 with a simple system where the firm produces and sells two complementary products as a set to the market. We then extend our model in Section 3 by considering a general assembly system where the firm needs to plan components production amounts and assembly amount sequentially with uncertain production and assembly capacities. Concluding remarks are given in Section 4. A symbol/notation table and all the detailed mathematical proofs are given in the appendixes.

2. System of two complementary products

2.1. Model assumptions

A firm produces and sells a set of two perfectly complementary products to the market in a selling season. Due to long lead time