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Innovative Applications of O.R.

Optimal transition to renewable energy with threshold of irreversible pollution

N. Bonneuil[a,*], R. Boucekkine[b,1]

[a] Institut national d'études démographiques, and École des hautes études en sciences sociales, France
[b] Université d'Aix-Marseille (Aix-Marseille School of Economics), Cnrs and Ehess, France

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ABSTRACT

When cheap fossil energy is polluting and pollutant no longer absorbed beyond a certain concentration, there is a moment when the introduction of a cleaner renewable energy, although onerous, is optimal with respect to inter-temporal utility. The cleaner technology is adopted either instantaneously or gradually at a controlled rate. The problem of optimum under viability constraints is 6-dimensional under a continuous-discrete dynamic controlled by energy consumption and investment into production of renewable energy. Viable optima are obtained either with gradual or with instantaneous adoption. A longer time horizon increases the probability of adoption of renewable energy and the time for starting this adoption. It also increases maximal utility and the probability to cross the threshold of irreversible pollution. Exploiting a renewable energy starts sooner when adoption is gradual rather than instantaneous. The shorter the period remaining after adoption until the time horizon, the higher the investment into renewable energy.

1. Introduction

Nordhaus (1973), Nordhaus (1991), Nordhaus (1992) pioneered equilibrium models of climate and economy. He linked global warming to carbon emissions and called for a transition to clean energy. Chakravorty, Moreaux, and Tidball (2008) and van der Ploeg and Withagen (2012) modelled what such a transition might be, with the characteristic that people would resort to renewable energy only when fossil resources were exhausted. In contrast, taking capital accumulation into account through capacity constraints and adjustment costs, both of which must be factored into the construction of renewable energy plants, Amigues, Ayong Le Kama, Chakravorty, and Moreaux (2013) and Lecuyer and Vogt-Schilb (2014) showed that optimal investment in renewable energy is independent of existing fossil resources. They also showed that it is better to start the transition as early as possible and to distribute investment over time rather than to wait and then be obliged to invest heavily over a short period.

We address the question of the best energy transition where a social planner takes the social cost of pollution into account, in contrast to what is done in the above-cited equilibrium models, where technological decisions are made by firms. We shall feature optimal transitions rather than mere equilibria, and tackle the difficulty of optimal timing: when should energy be switched from fossil to renewable? Our perspective resembles that of Tahvonen and Withagen (1996), Tahvonen (1997), Valente (2011), Prieur, Tidball, and Withagen (2013), and Boucekkine, Pommeret, and Prieur (2013a), Boucekkine, Pommeret, and Prieur (2013b).

Amigues et al. (2013) identified the various optimal regimes, paying considerable attention to state constraints, but ignoring timing controls. We do consider timing controls and the possibility that adoption of renewable energy is gradual rather than instantaneous. Boucekkine et al. (2013a) used timing controls in multi-stage optimal control; Bonneuil and Saint-Pierre (2008) introduced timing controls in the economic theory of the life cycle. Our original way of involving timing controls in a set-valued dynamic under state and control constraints allows us to avoid the algebraic complexity associated with the consideration of state constraints on capital. The problem of energy transition has also been addressed in operation research, through multi-criteria analysis (Georgopoulou, Sarafidis, Mirasgedis, Zaimi, & Lalas, 2003; Kowalski, Stagl, Madlener, & Omann, 2009). When uncertainty is at stake, Lukas and Welling (2014) took the real option set-up. Our handling of explicit timing controls and optimal regime switching is closer to the impulse control technique, as in Hainaut (2014), who studies pension funds in a regime switching economy. We differ from this author in our use of set-valued analysis, which allows us to deal with uncertainty, state constraints, and impulse controls. We also differ from him by the fact that we develop

* Corresponding author. Tel.: +33 156 062 138.
E-mail addresses: bonneuil@ined.fr (N. Bonneuil), raouf.BOUCEKKINE@univ-amu.fr (R. Boucekkine).

1 Senior member, Institut universitaire de France

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an econometric part to test the implications and the robustness of the results.

With regard to energy, the share of renewable energy in French electricity production, for example, is forecast to grow from 22.9% in 2013 to 35% in 2020.[2] We propose to determine not only when the transition to a renewable energy should start, but also at what rate it should replace existing forms of energy. The proportion of clean but expensive energy can then be adjusted over time.

We consider pollution as possibly irreversible, as Tahvonen and Withagen (1996) and Prieur (2009) did. The non-renewable resource, say fossil fuels, is extracted and consumed, causing emissions of pollutant. Beyond a critical concentration, pollutant is no longer absorbed, as it is the case for carbon dioxide by the oceans. The choice is to adopt a renewable but expensive technology, say wind power, or not. Prieur et al. (2013) searched for the optimal management of exhaustible resources under irreversible pollution but ignored the adoption of the new technology. In Boucekkine et al. (2013b), the timing of adoption is endogenous but not the size of the investment in the technology of renewable energy. This assumption follows Valente (2011): when adoption starts, the share of renewable energy is set to a constant, reflecting the maximal level of the renewable energy allowed by the technological capacity of the country. We refer to this type of adoption as “instantaneous adoption.” We relax the view of instantaneous adoption, which is mathematically convenient but restrictive, and suggest to endogenize the investment into the renewable energy, within the possibilities left by the technology. We refer to this type of adoption as “gradual adoption.”

Boucekkine et al. (2013b), using Pontryagin, had to check all possible candidates to optimality (inner solutions, corner solutions, solutions with adoption of renewable energy and those without, solutions with reversibility of pollution and those without), computing the associated value functions for each set of parameters and for each set of initial conditions, to finally select the maximal value. In contrast, viability theory yields optima in a necessary and sufficient way, allowing a systematic exploration of the state space. Our method here allows one to solve the problem without the round-about of Pontryagin or Hamilton-Jacobi-Bellman, without having to compute each candidate solution to select the best one thereafter. Our econometric analysis, as in Bonneuil and Boucekkine (2014), shall capture the main features of viable optimal decisions and help us interpret them. Varying the production unit cost, the maximal investment into renewable energy, and the threshold of irreversible pollution allows us to test robustness on the parameters setting. Besides, thanks to Bonneuil’s 2006 viability algorithm, the technique is flexible enough to add state dimensions, here gradual versus instantaneous adoption, avoiding to derive first-order conditions each time. The technique works with continuous-discrete time dynamics. The link with optimal solutions, which economists are fond of, was established theoretically (Bonneuil, 2012). It finds here a practical application.

Around the plausible parameters values used by Prieur et al. (2013), we shall determine the factors favoring adoption or not. We shall notably find the importance of initial fossil resource and initial level of pollution in the maximal inter-temporal utility. This is consistent with Boucekkine et al. (2013b), who use a different analysis. In addition, we shall also examine the role of the time horizon and the capacity to afford a gradual adoption. We shall also find that a longer time horizon increases the probability of adoption of renewable energy and the time for this adoption. It also increases maximal utility and the probability to cross the threshold of irreversible pollution. We shall find that the time for starting exploiting a renewable energy decreases when adoption is gradual, compared with instantaneous adoption: the shorter the time remaining after adoption until the time horizon, the more quickly renewable energy is adopted. Then the theory of optimal regime switching is completed by the realistic feature of gradual adoption, a key component of energy policies.

After posing the problem as a maximization under four differential equations, we shall present viability theory and the procedure to obtain a maximum under viability constraints, with its associated algorithm. The problem becomes a viability problem with six dimensions. An example of trajectories with gradual or with instantaneous adoption will helps us situate the dynamic. Then we shall proceed to the econometric analysis of 600 simulations, so as to highlight the determinants of adoption and its mode.

2. The problem

The quantity of fossil resource is $x_1(t)$ at time t. People are to solve the trade-off between cheap polluting energy against expensive cleaner energy, through maximizing the discounted stream of utilities from energy consumption and disutilities associated with pollution and the cost of investing into renewable energy:

$$\max_{v_1, v_2, t_s} \int_0^T \Big(u(v_1(t)) - D(x_2(t)) - cx_3(t)\Big)e^{-\delta t}\, dt, \tag{1}$$

where T is the time horizon, the function u represents utility and D is the damage function, c is a parameter reflecting the production unit cost, x_2 is the level of pollution, x_3 is the amount of exploited renewable energy, $t_s \geq 0$ is the time when this renewable resource begins to be adopted:

$$\begin{cases} x_3'(t) = 0 \quad \text{if } t < t_s \\ x_3'(t) = v_2(t) \in V_2 \quad \text{if } t \geq t_s \text{ (adoption)} \\ \quad x_3 \in [0, \bar{x}_3], \end{cases} \tag{2}$$

where V_2 is a closed set, v_2 is the investment into or disinvestment from renewable energy, $\bar{x}_3$ is the maximal renewable energy, limited by the possibilities of the country. Total energy consumption $v_1(t)$ is the sum of the quantity $e(t) \geq 0$ of polluting energy and of the quantity $x_3(t) \geq 0$ of renewable non-polluting energy:

$$v_1(t) = e(t) + x_3(t). \tag{3}$$

The fossil resource decreases as:

$$x_1'(t) = -e(t) = -v_1(t) + x_3(t). \tag{4}$$

The quantity of pollutant is also taken equal to $e(t)$, such that pollution varies according to:

$$x_2'(t) = \max(0, v_1(t) - x_3(t)) - \alpha(t)x_2(t), \tag{5}$$

where $\alpha(t)$ is the rate of absorption of pollution by the environment, becoming null over a threshold value $\underline{x}_2$, above which the milieu becomes unable to absorb any quantity of pollutant:

$$\begin{cases} \alpha(t) = \alpha \quad \text{constant for } x_2(t) \leq \underline{x}_2 \quad \text{(reversibility)} \\ \alpha(t) = 0 \quad \text{otherwise.} \quad \text{(irreversibility).} \end{cases} \tag{6}$$

Equation (2) is an impulse equation:

$$\begin{cases} x_3'(t) = w(t)v_2(t) \\ w'(t) = 0, \quad w(0) = 0 \\ w(t_s^+) = w(t_s^-) + 1 = 1 \\ \quad x_3 \in [0, \bar{x}_3], \end{cases} \tag{7}$$

where w is a Heaviside function.

The dynamic (4–7) is a differential inclusion with impulse:

$$\begin{cases} x'(t) \in F(x(t)) \\ \quad x^+ = R(x) := \{x_1^-, x_2^-, x_3^-, 1)\} \end{cases} \tag{8}$$

[2] Source *Réseau de transport d'électricité*, published by Bertille Bayart in *Le Figaro*, 10 December 2013.