European Journal of Operational Research 245 (2015) 203–208

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Congestion measurement in nonparametric analysis under the weakly disposable technology

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ARTICLE INFO

Article history:
Received 29 April 2014
Accepted 2 March 2015
Available online 5 March 2015

Keywords:
Data envelopment analysis
Undesirable output
Congestion

ABSTRACT

Congestion is a widely observed economic phenomenon where outputs are reduced due to excessive amount of inputs. The previous approaches to identify congestion in nonparametric analysis only consider desirable outputs. In the production process, undesirable outputs are usually jointly produced with desirable outputs. In this paper, we propose an approach for measuring congestion in the presence of desirable and undesirable outputs simultaneously. The proposed approach can discriminate between the congested DMUs and the truly efficient DMUs, which are all efficient according to the scores calculated by the directional distance function. Finally, an empirical example is used to illustrate the approach.

1. Introduction

The concept of congestion, first introduced by Färe and Svensson (1980), is a widely phenomenon where excessive amounts of the input cause a reduction of the output. Subsequently, it was extended and developed by Färe, Grosskopf, and Lovell (1985), Cooper, Thompson, and Thrall (1996), and Cooper, Seiford, and Zhu (2000) in the context of DEA (data envelopment analysis). Since then, the treatment of congestion within the DEA framework has received considerable attention and several approaches have been proposed to identify congestion (Cherchye, Kuosmanen, & Post, 2001; Cooper, Gu, & Li, 2001a; Färe & Grosskopf, 2000; Kao, 2010; Khoveyni, Eslami, khodabakhshi, Jahanshahloo, & Hosseinzadeh Lotfi, 2013; Sueyoshi & Sekitani, 2009; Tone & Sahoo, 2004).

Färe et al. (1985) proposed a radial-model approach in which congestion is measured as the difference between technologies under weak and strong disposability inputs. Cooper et al. (1996) proposed a slack-based approach, where the congestion effect is measured as the difference between the observed amounts and the expected amounts. Cooper et al. (2001a) compared the above approaches and claimed that the approach by Färe et al. (1985) can fail to identify congestion in some situations. See some debates on the subject of congestion (Cherchye et al., 2001; Cooper, Gu, & Li, 2001b; Färe & Grosskopf, 2000).

Further, Tone and Sahoo (2004) provided a theoretical linkage between congestion and returns to scale (RTS). Moreover, their approach can detect the strong and weak congestion status. However, Tone and Sahoo (2004) implicitly assume a unique optimal solution in the investigation on DEA-based congestion. In the presence of multiple solutions in the congestion measurement, the economic implications of congestion obtained by Tone and Sahoo (2004) are all problematic from both theoretical and practical perspectives. To deal with the issue, Sueyoshi and Sekitani (2009) proposed an analytical approach to handle an occurrence of multiple solutions and measure the degree of wide congestion.

However, all the previous approaches on congestion only consider desirable outputs. In the production process, undesirable outputs are usually jointly produced with desirable outputs. Therefore, a new framework for measuring congestion should be developed in the presence of desirable and undesirable outputs simultaneously. A pioneering paper by Färe, Grosskopf, Lovell, and Pasurka (1989) considers undesirable outputs to be weakly disposable, which means that a reduction in the good outputs should result in an equiproportionate reduction of the undesirable outputs (Chung, Färe, & Grosskopf, 1997; Färe & Grosskopf, 2003, 2004, 2009; Kuosmanen, 2005; Kuosmanen & Kortelainen, 2005; Kuosmanen & Matin, 2011; Kuosmanen & Podinovski, 2009; Picazo-Tadeo, Beltrán-Esteve, & Gómez-Limón, 2012; Weber & Domazlicky, 2001; Zhou, Ang, & Poh, 2008). Treating undesirable outputs in their original forms with the assumption of weak-disposability is consistent with the physical laws and the standard axioms of production theory (Färe & Grosskopf, 2003, 2004, 2009). Based on the weak disposable technology, many empirical studies utilized the directional distance function model (Chung et al., 1997), which expands the desirable outputs and contracts inputs and undesirable outputs along the direction vector path to assess the efficiency.

In this paper, based on the directional distance function, we develop an approach to identify the occurrence of congestion

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http://dx.doi.org/10.1016/j.ejor.2015.03.001

(strong and weak) in the presence of desirable and undesirable outputs simultaneously. Different from the traditional circumstance with desirable outputs only, we find that even if a DMU is efficient by the directional distance function, it maybe evidences congestion when considering both desirable outputs and undesirable outputs. Through our proposed approach, we can discriminate between the congested DMUs and the truly efficient DMUs, which are all efficient according to the scores calculated by the directional distance function.

The remaining structure of this research is organized as follows: In Section 2, the concepts of strong and weak congestions in the presence of desirable and undesirable outputs are defined. Section 3 proposes an approach to identify the occurrence of strong and weak congestions. Section 4 compares the proposed approach with the existing three representative approaches and applies the proposed approach to analyze an empirical dataset consisting of 20 power plants. Section 5 concludes the paper.

2. Preliminaries

Assume that there are K units and each unit uses a vector of inputs $x \in \mathbf{R}_+^N$ to produce a vector of good outputs $y \in \mathbf{R}_+^M$ and bad outputs $b \in \mathbf{R}_+^I$. The production technology consisting of all feasible (x, y, b) can be defined by:

$$\Omega = \{(x, y, b)\,|x \text{ can produce } (y, b)\} \tag{1}$$

Given that we consider K observed DMUs, the production technology set can be formulated as follows:

$$\Omega = \left\{ \begin{array}{ll} (x, y, b) : \theta \sum_{k=1}^{K} z_k y_{km} \geq y_m & m = 1, \cdots, M \\ \theta \sum_{k=1}^{K} z_k b_{ki} = b_i & i = 1, \cdots, I \\ \sum_{k=1}^{K} z_k x_{kn} \leq x_n & n = 1, \cdots, N \end{array} \right\} \tag{2}$$

By considering a sample of K observed DMUs, inefficiency for unit k_o exhibiting constant returns to scale and weak disposability can be computed by the following direction distance function (Chung et al., 1997):

$$IE(k_o) = \max\ \delta \tag{3}$$

$$\begin{array}{ll} \sum_{k=1}^{K} z_k y_{km} \geq y_{k_o m} + \delta y_{k_o m} & m = 1, \cdots, M \\ \sum_{k=1}^{K} z_k b_{ki} = b_{k_o i} - \delta b_{k_o i} & i = 1, \cdots, I \\ \sum_{k=1}^{K} z_k x_{kn} \leq x_{k_o n} - \delta x_{k_o n} & n = 1, \cdots, N \\ z_k \geq 0 & k = 1, \cdots, K \end{array}$$

where $z = (z_1, \cdots, z_K)$ are referred to the intensity variables and $g = (x_{k_o n}, y_{k_o m}, b_{k_o i})$ is the direction vector. The less $IE(k_o)$ is, the more efficient k_o is. If $IE(k_o) = 0$, unit k_o is efficient. Otherwise, it is inefficient. If unit k_o is inefficient, make a projection in the following manner:

$$x'_{k_o} = x_{k_o} - \delta^* x_{k_o}, \quad y'_{k_o} = y_{k_o} + \delta^* y_{k_o} \quad \text{and} \quad b'_{k_o} = b_{k_o} - \delta^* b_{k_o}. \tag{4}$$

The projected point $(x'_{k_0}, y'_{k_0}, b'_{k_0})$ is efficient with respect to Ω.

In the following, we use a simple example to illustrate the drawback of model (3). Table 1 shows the data set of five DMUs with two inputs (x_1 and x_2), two desirable outputs (y_1 and y_2) and two undesirable outputs (b_1 and b_2).

The inefficiency scores by model (3) are listed in the last column of Table 1. According to the inefficiency scores in the last column, all DMUs are efficient. However, from DMU2 to DMU3, a phenomenon of congestion has occurred because the desirable output decreases and both undesirable outputs increase as the input increases.

Table 1
The data set for illustration.

	x_1	x_2	y_1	y_2	b_1	b_2	Inefficiency
DMU1	1	3	2	8	3	2	0
DMU2	1	4	3	7	2.5	4	0
DMU3	2	6	1	6	5	35	0
DMU4	3	6	1	5	10	10	0
DMU5	1.5	3	1.5	8	3	3	0

Remark. According to Brockett, Cooper, Deng, Golden, and Ruefli (2004), congestion is often referred to as a "particularly severe form of inefficiency" in terms of economics. A DMU evidences congestion if and only if it is not weakly efficient by DEA models when considering desirable outputs only (Wei & Yan, 2004). However, when considering both desirable and undesirable outputs, even if a DMU is efficient, it maybe evidences congestion.

In the real world, undesirable outputs such as smoke pollution or waste are unavoidably generated along with desirable outputs. Thus, in the above scenario, the outputs are divided into two categories, desirable and undesirable. For desirable outputs, the more the value is, the better the performance is while for undesirable outputs, the less the value is, the better performance is. Therefore, similar to Tone and Sahoo (2004), we first define the concepts of "strong congestion" in the presence of desirable and undesirable outputs:

Definition 1. A DMU$_k$ (x_k, y_k, b_k) is "strongly congested" if it is efficient and there exists an activity $(\tilde{x}, \tilde{y}, \tilde{b}) \in \Omega$ such that $\tilde{x} = \alpha x_k$ (with $0 < \alpha < 1$), $\tilde{y} = \beta y_k$ (with $\beta > 1$) and $\tilde{b} = \gamma b_k$ (with $0 < \gamma < 1$).

The above definition means that a DMU$_k$ (x_k, y_k, b_k) is in the status of strong congestion requires that a proportionate reduction in all inputs can give rise to an increase in all desirable outputs and a decrease in all desirable outputs. From this viewpoint, Definition 1 is too restrictive in some cases. In the following, we define the concept of "weak congestion" by relaxing such stringent requirements.

Definition 2. A DMU$_k$ (x_k, y_k, b_k) is "weakly congested" if it is efficient and there exists an activity that uses less resources in one or more inputs to produce more products in one or more desirable outputs and less undesirable outputs in one or more undesirable outputs.

Note that strong congestion implies weak congestion but not vice versa. In a single input, a single desirable output and an undesirable output case, there is no distinction between strong and weak congestions.

3. Proposed approach

In this section, we proposed an approach to identify the occurrence of congestion. By making use of the duality theory of linear programming, the dual formulation of the direction distance model (3) is described as follows:

$$\text{Min} \quad b_{k_0}\pi^b + x_{k_0}\pi^x - y_{k_0}\pi^y \tag{5}$$

$$\text{s.t.} \quad y_k\pi^y - b_k\pi^b - x_k\pi^x \leq 0 \quad k = 1, \cdots, K \tag{5.1}$$

$$\begin{array}{l} b_{k_0}\pi^b + x_{k_0}\pi^x + y_{k_0}\pi^y = 1 \\ \pi^x \geq 0 \\ \pi^y \geq 0 \\ \pi^b \text{ unconstrained} \end{array} \tag{5.2}$$

Let π^{x*}, π^{y*} and π^{b*} be the optimal solution to the model (4).

Theorem 1. *A DMU$_{k_0}$ $(x_{k_0}, y_{k_0}, b_{k_0})$ is in the status of strong congestion if and only if for at least one $i \in \{1, \cdots I\}$, π_i^{b*} is negative.*