European Journal of Operational Research 232 (2014) 156–168

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R

On the decisiveness of a game in a tournament

Gery Geenens *

School of Mathematics and Statistics, University of New South Wales, Sydney, NSW 2052, Australia

ARTICLE INFO

Article history:
Received 27 December 2012
Accepted 17 June 2013
Available online 27 June 2013

Keywords:
OR in sports
Applied probability
Uncertainty modelling
Game importance
Entropy

ABSTRACT

In sport tournaments in which teams are matched two at a time, it is useful for a variety of reasons to be able to quantify how important a particular game is. The need for such quantitative information has been addressed in the literature by several more or less simple measures of game importance. In this paper, we point out some of the drawbacks of those measures and we propose a different approach, which rather targets how decisive a game is with respect to the final victory. We give a definition of this idea of game decisiveness in terms of the uncertainty about the eventual winner prevailing in the tournament at the time of the game. As this uncertainty is strongly related to the notion of entropy of a probability distribution, our decisiveness measure is based on entropy-related concepts. We study the suggested decisiveness measure on two real tournaments, the 1988 NBA Championship Series and the UEFA 2012 European Football Championship (Euro 2012), and we show how well it agrees with what intuition suggests. Finally, we also use our decisiveness measure to objectively analyse the recent UEFA decision to expand the European Football Championship from 16 to 24 nations in the future, in terms of the overall attractiveness of the competition.

1. Introduction

Nowadays, major sport events regulate the life style of many people. Supporters, communities or even whole countries often stop any other activities when their teams are playing. All over the world, thousands of people regularly go to stadiums to attend sport competitions, while millions more follow them on television. It seems clear, however, that the popular craze for a given contest or game would depend on its importance. For instance, take the case of a tournament in which teams (or individual contestants) are matched two at a time, like in football (*soccer*), basketball or tennis. It seems natural to expect popular passion to be higher around games which might be decisive as to the final winner of the tournament, than around games which cannot dramatically change the eventual tournament outcome. In this paper, we define and investigate that notion of 'decisiveness' of a game in a tournament organized as described above.

Comprehending and quantifying how decisive a game is within a tournament is important for many purposes. A game decisiveness measure should be a paramount component in a model aiming to explain the game attendance or television audience (Downward & Dawson, 2000; Jennett, 1984; Villa, Molina, & Fried, 2011), or even the game outcome (Audas, Dobson, & Goddard, 2002). Shortly before the game, its decisiveness can also motivate the referee assignment, with the best referees assigned to the most decisive games. At the team level, it could also influence the team lines-up, as coaches may be tempted to experiment with new tactics on non-decisive games whose outcomes do not matter.

More generally, the overall attractiveness of a competition can be quantified through the number of games expected to be decisive. A competition with very few potentially decisive games, with an eventual winner known very early in the competition, would not be very attractive to the public, unlike a competition with numerous potential twists and turns (Goossens, Beliën, & Spieksma, 2012). This observation can be important at the time of deciding about the design of a tournament (Scarf, Mat Yusof, & Bilbao, 2009), that is, parameters such as the number of teams competing, the schedule, the rules for ranking, and qualification/elimination or promotion/relegation when applicable. Given that a competition format with more decisive games would definitely be more attractive for viewers, increasing that overall decisiveness of games could then be a driving factor. Cairns (1987), Bojke (2007), Goossens et al. (2012) investigated that concept. Finally, for television broadcasters, decisiveness must also be an important element when making decisions about which games to broadcast (Forrest, Simmons, & Buraimo, 2005), as games with low stake do not attract viewers. Given the importance of television in today's sporting events, in terms of broadcasting contract and advertisement return, this is also an important fact to take into account.

Incidentally, this makes it clear that one should be able to assess the decisiveness of a game well ahead of time, in particular with some other games still to be played before that particular game. Indeed, television stations usually have to choose their games for

* Tel.: +61 2 938 57032; fax: +61 2 9385 7123.
E-mail address: ggeenens@unsw.edu.au

0377-2217/$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2013.06.025

broadcast in advance, and sometimes even before the tournament starts, in the case of major tournaments like the football FIFA World Cup or UEFA European Championship. In such a situation, the effective decisiveness of the game on the day will depend on the uncertain outcome of other games yet to be played, and is thus essentially random. We will then be talking about the *expected decisiveness* of a game. This expectation is to be defined with respect to a joint probability distribution for the outcomes of the games remaining to play, that should be properly devised. Estimating the probabilities of seeing a team winning or losing (or drawing, if allowed) on a given game has been the topic of a huge literature in sports statistics. In particular, for the case of football, various models were investigated in Reep and Benjamin, 1968, Maher (1982), Baxter and Stevenson (1988), Kuk (1995), Dixon and Coles (1997, 1998), Dyte and Clarke (2000), Karlis and Ntzoufras (2003, 2009), Tena and Forrest (2007), Brillinger (2008), Geenens (2010), Flores, Forrest, and Tena (2012). Assessing and analysing those probabilities is not the purpose of this paper, though. Here, we will just make use of previously derived outcome distributions or models when we will need them, and we will therefore assume that the probability distribution of outcomes for games to come is known.

Now, the way the tournament is organized (what we called above the tournament design) is also expected to play a paramount role in the decisiveness of any of its game. Scarf et al. (2009), Scarf and Mat Yusof (2011) defined two fundamental tournament designs. The first one is the classical *round-robin*: each competing team plays every other team a fixed number of times, earning points according to their results (in football, typically a win is worth 3 points and a draw is worth 1 point), and the winning team is the one with the largest number of points at the end of the tournament (with particular rules to break ties). Most of the major national football leagues in Europe (e.g. English Premier League, Spanish Liga, German Bundesliga) are played according to this design, with each team playing all the others twice, once at their home field and once away. The second fundamental tournament design is the *knock-out*: games are played in round, and only competitors winning in round k take part in round $k + 1$, the others being definitely eliminated. At the end, the winner is the winning competitor at the final round, that is, the only undefeated competitor. Grand slam tennis tournaments are typically built according to this design. In football this is also the case for most of the European domestic cup competitions, like the celebrated FA Cup in England.

Many other major sport competitions are designed as hybrid between these two fundamental structures. So, the FIFA World Cup is currently organized in two stages: the first one being 8 parallel simple round-robins (called ‘groups’), each matching 4 teams with the best two progressing to the second stage, this one being a knock-out tournament starting with the 16 teams qualified from the first stage. Between 1996 and 2012, the UEFA European Championship was a similar two-stage tournament, except that only 16 teams took part. The first stage therefore consisted in 4 groups, of which the best 8 teams (2 per group) go to play the second knock-out stage. It has, however, been announced (Union of European Football Associations, 2008) that from 2016, 24 teams would take part in the final tournament, which has attracted a lot of criticism. Detractors claim that this will lower the level of the tournament and decrease its intensity. In Section 4, we investigate how this fundamental change in the tournament design would affect the decisiveness of its games and consequently its overall attractiveness.

The paper is organized as follows. In Section 2, we review the related notion of game importance and we point out some important drawbacks of it. These justify the definition of a different quantity to measure how decisive a game is/may be. We define such a game decisiveness measure in Section 3 and show how it addresses the shortcomings of the game importance measure previously proposed in the literature. In Section 4 we investigate further that notion of game decisiveness in practice on two real competitions: the 1988 NBA Championship Series, also considered in Schilling (1994), and the UEFA European Championship 2012. Section 5 concludes.

2. Game importance measures

The notion of *decisiveness* of a game, as we think of it, is different to the concept of how *important* a game is in a tournament as it was addressed previously in the literature. Among others, some of the works cited in Section 1 use various simple game importance measures in their respective framework. In the case of a football tournament for instance, Audas et al. (2002) call a game important if it is still possible at the time of the game for either of the opponents to win the tournament, assuming that all other teams will draw in the rest of their games. With that definition, game importance is binary, and most certainly lacks of nuance. Jennett (1984) measures the importance of a game for a competing team as the inverse of the number of games they have still to win to be crowned winner; whereas, Downward and Dawson (2000) suggest a slight refinement of the previous idea making use of the number of extra points necessary for a team to win the tournament. Goossens et al. (2012) define the importance of a match m as

$$I_m = \frac{2n_m}{A_m^2},$$

where A_m is the total number of teams still competing for the final victory of the tournament at the time of the match, and n_m is the number of teams in match m still in competition ($n_m = 0$, 1 or 2). A drawback of this is that it does not take into account the temporal position of the game in the tournament. Specifically, consider the game matching the last two teams remaining in competition, which should indeed be quite important. Then, $I_m = 1$, but this at any time in the competition. However, this game should be much more important if it was one of the very last of the competition, than if many other games were left for those two teams, with many occasions to catch up.

Also, it does not take into account the strength of the matching teams. At the same stage of the competition (thus with the same number A_m), a game matching two underdogs still in competition (but if early enough in the tournament, all teams are usually still alive) would be as important as a game matching the greatest two favourites of the competition. It seems, however, clear that the latter is more crucial than the former. Scarf and Shi (2008) reflects this by saying that some matches may only be important “on paper”, meaning that even if the two competing teams are still technically in competition at the time of the match, it is very unlikely that they will really be running for the final victory until the end of the tournament. This is actually where ideas based on probabilities come in. It turns out that the most widely used probability-based importance measure for a game so far has been the *Schilling importance*.

Consider a tournament made up of n games altogether. The Schilling importance (Scarf & Shi, 2008; Schilling, 1994) of game k ($k = 1, \ldots, n$) for team i when assessed right after game k_0 ($k_0 = 0, \ldots, k-1$) is defined as

$$S_{i,k_0,k} = \mathbb{P}(V_i|W_{i,k}, H_{k_0}) - \mathbb{P}(V_i|L_{i,k}, H_{k_0}),$$

where V_i is the event that team i is the eventual winner of the tournament, $W_{i,k}$ and $L_{i,k}$ are the events that the outcome of game k is favourable or unfavourable to team i, and H_{k_0} represents the whole history of all matches played in the tournament up to and including game k_0. Note that we define the case $k_0 = 0$ as the prior situation, that is, right before the tournament starts. The importance of game