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Decision Support

Non-cooperative and cooperative game-theoretic models for usage-based lease contracts

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ABSTRACT

In this paper, we study game-theoretic models for lease contracts, by which the owner (lessor) rents a piece of equipment to a user (lessee). The lessee decides on the optimal lease period and usage rate, and the lessor is responsible for developing a maintenance policy for the equipment. Two non-cooperative game-theoretic models and a cooperative model are developed to describe the relationships between the two decision makers. In the non-cooperative simultaneous move game, the lessee and the lessor act simultaneously and independently to make their decisions. In the leader–follower non-cooperative game, the lessor is the leader who specifies the maintenance policy first, and the lessee, as the follower, decides on the lease period and usage rate accordingly. For these two games, the Nash and Stackelberg equilibria are obtained respectively. For the cooperative game, we derive the solution targeting on total profit maximization and show that this solution can be implemented as an equilibrium using a nonlinear transfer-payment contract. Besides, we compare the Nash equilibrium, Stackelberg equilibrium, and the total maximum solution to each other, and our results show that the lessee and lessor can gain more profit from the cooperative contract than from the non-cooperative alternatives. Numerical examples are provided to demonstrate the different solution methodologies and the value of cooperation.

1. Introduction

Equipment leasing is a process by which the owner (lessor) rents a piece of equipment to a user (lessee) under a lease contract. It is increasingly common for companies to lease equipment rather than to own it (Jaturonnatee, Murthy, & Boondiskulchok, 2006). Over 80% of American businesses lease at least one of their equipment acquisitions, and nearly 90% say they would choose to lease again (Giddy, 2004). Part of the motivations is saving on initial investment, flexibility on equipment upgrading, and cost reduction in maintenance and inventory. In this paper, we study a lease contracting problem where the lessee decides on the optimal lease period and usage rate and the lessor is responsible for developing a maintenance policy for the equipment.

Many studies on leased equipment deterioration are focused only on the effect of age (time) while ignoring the effect of actual usage. In reality, however, many lease contracts are characterized by both the lease period (time) and the usage. A typical example is an automobile leased for 3 years and up to 10,000 miles per year. Other examples of usage include the number of pages produced by a photocopier, the flight hours operated by aircraft (Jack, Iskandar, & Murthy, 2009), and the weight carried by a mining haul truck. Although extensive research has been done on warranty policies considering both time and usage (Chen & Popova, 2002; Iskandar, Murthy, & Jack, 2005; Jack et al., 2009; Kim & Rao, 2000; Shafiee & Chukova, 2013), they have received little attention in decision making of lease contracts. In this paper, the effects of both time and usage on deterioration of leased equipment are considered.

In practice, preventive maintenance (PM) is usually performed by the lessor to achieve a trade-off between the failure cost and the PM cost. In the literature, PM policies considering imperfect maintenance of leased equipment have been studied. Jaturonnatee et al. (2006) derived the optimal number of PM actions to be carried out during the lease period along with the time and the degree of each PM action. Conceptually, the degree of PM quantifies the reduction in the equipment's failure intensity. Yeh, Kao, and Chang (2009) considered a problem assuming a fixed degree for all maintenance actions. Pongpech and Murthy (2006) determined the degree of each action under a periodic maintenance policy. Chang and Lo (2011) studied a case where PM actions were carried out

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Nomenclature

PM	Preventive maintenance
CM	Corrective maintenance
r	Usage rate (a decision variable of the lessee)
K	Lease period (a decision variable of the lessee)
N	Number of PM actions (a decision variable of the lessor)
δ	Degree of each PM (a decision variable of the lessor)
L	Life cycle of the equipment
r_m	Maximum usage rate
$\lambda_0(t)$	Failure intensity function with no PM
$\lambda(t)$	Failure intensity function with PM actions
$H(\mathbf{x})$	Nonlinear transfer function
$\gamma, \bar{\alpha}_0, \beta$	Side-payment contract parameters
Π_1	Lessee's payoff
Π_2	Lessor's payoff
$\bar{\Pi}_1$	Lessee's payoff with transfer payment
$\bar{\Pi}_2$	Lessor's payoff with transfer payment
Π	Total payoffs of lessee and lessor
$\mathbf{x}_n^*$	Nash equilibrium
$\mathbf{x}_s^*$	Stackelberg equilibrium
$\mathbf{x}_c^*$	Total maximum solution (cooperative)

when the equipment's failure intensity reached a specified level. In this paper, we assume that the lessor will perform periodic PM (Yeh & Chen, 2006) with a fixed degree for all actions. Such PM actions not only reduce the number of failures during the lease period but also increase the residual value of the equipment by the time when the lease contract expires. Besides, we consider that the productivity of the leased equipment decreases as it deteriorates, so PM actions also reduce the revenue loss caused by deterioration (Wu, Xie, & Ng, 2011).

A lease contract should be an instruction that is satisfactory to both lessor and lessee (Wang, Wallace, Shen, & Choi, 2015). To this end, developing a framework to address their individual perspectives is crucial (Murthy & Jack, 2014). One of the most popular approaches in dealing with such problems is game theory. Depending on the behavior of the decision makers (players), game-theoretic models can be classified into non-cooperative and cooperative games. In a non-cooperative formulation, players decide independently. Nash (Dong, Zhang, & Nagurney, 2004) and Stackelberg equilibria (Esmaeili, Aryanezhad, & Zeephongsekul, 2009) are the two most popular solution concepts used in these games, both of which are based on the analysis of best response functions. When the players choose their strategies simultaneously, Nash equilibrium applies, but when one player decides before the other, the Stackelberg solution is appropriate. The advantage of contracting based on these equilibrium solutions is that no player has a temptation to deviate from the agreement to increase his profit (Chiu, Choi, Li, & Xu, 2014). However, these contracts are not efficient since they do not maximize the total profit of the players (Cachon, 2003). A possible solution is for the decision makers to cooperate when determining the terms of contract (Giannoccaro & Pontrandolfo, 2004; Leng & Parlar, 2005; Matsumoto & Szidarovszky, 2016), so that an outcome better than the Nash or Stackelberg equilibria (Kim & Ha, 2003; Nagarajan & Sošić, 2008) can be achieved. Esmaeili, Gamchi, and Asgharizadeh (2014) presented three-level warranty service contracts among manufacturer, agent, and customer. Under different game-theoretic games, they determined the optimal sale price, warranty period, and warranty price for the manufacturer and the optimal maintenance cost for the agent by maximizing their profits.

In this paper, we study three lease contract models based on non-cooperative and cooperative games. For the non-cooperative games, two scenarios are considered: non-cooperative simultaneous move game and non-cooperative leader–follower game. In the first scenario, the lessee and lessor choose their strategies simultaneously while, in the second scenario, the lessor dominates the lessee by determining his maintenance policy first, and then the lessee chooses the lease period and usage rate. We respectively derive the Nash and Stackelberg equilibria and compare the corresponding strategies and payoffs. Next, for the cooperative game model, we derive the solution targeting on total profit maximization. It is of interest to compare the performance of the non-cooperative contracts and the cooperative alternative. One measure of performance is the difference between the total profit of a non-cooperative contract and that of a cooperative one (which has the maximum total profit). By comparing the total profits, we will demonstrate the conditions under which the non-cooperative lease contracts have a poor performance, and thus the lessee and lessor can gain much more profits by switching to the cooperative alternative.

It is worth pointing out that implementation of the cooperative solution requires two major criteria. First, cooperation should lead to a win–win situation for the lessee and lessor (Tarakci, Tang, Moskowitz, & Plante, 2006), such that the profits of both players become higher compared to the non-cooperative cases. Second, the players should have no incentive to deviate from the cooperative solution, i.e., they should modify their profits such that the total maximum solution becomes identical to a Nash equilibrium (for the simultaneous move case) and a Stackelberg equilibrium (for the leader–follower case) (Cachon & Zipkin, 1999). We will show that these two criteria can be achieved by a nonlinear transfer-payment contract. We have allocated the maximum total profit between the players based on Nash bargaining solution to satisfy the first criterion. The related literature has overwhelmingly showed cases where decision makers negotiate over different contract terms (Nagarajan & Sošić, 2008). Gurnani and Shi (2006) and Nagarajan and Sošić (2008) provided reviews of such contracts. Bajari, McMillan, and Tadelis (2009) analyzed a comprehensive data set of building construction contracts and observed that almost half of the contracts were developed through negotiation. The study suggested that more complicated projects were more likely to be negotiated. Similar approaches have been considered in distribution channels, franchising arrangements, and inventory control systems (Chen, Federgruen, & Zheng, 2001; Leng & Parlar, 2010; Leng & Zhu, 2009; Nagarajan & Bassok, 2008).

The remainder of this paper is organized as follows. The mathematical formulations for equipment failure intensity, maintenance costs, revenue, residual value, as well as the lessee's and the lessor's payoff functions are given in Section 2. The non-cooperative solution methodologies are described in Section 3, where the best response functions of lessee and lessor are derived, the Nash and Stackelberg equilibria are determined, and the corresponding strategies and payoffs are compared. Section 4 analyzes the cooperative game solution and compares it with the non-cooperative solutions. Next, a nonlinear transfer-payment function is introduced to implement the cooperative solution. Section 5 presents numerical examples to illustrate the three lease contract models and to investigate their performance. Finally, Section 6 concludes the paper.

2. Problem description and model formulation

It is assumed that the new equipment to be leased has life cycle L, and its maximum usage rate is r_m (Iskandar et al., 2005; Jack et al., 2009) which is the maximum allowable capacity per unit of time (e.g., 20,000 miles per year for a leased car or 240 tons per