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Decision Support

A new model for intuitionistic fuzzy multi-attributes decision making

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ABSTRACT

In this study, we discuss linear orders of intuitionistic fuzzy values (IFVs). Then we introduce an intuitionistic fuzzy weighted arithmetic average operator. Some fundamental properties of this operator are investigated. Based on the introduced operator, we propose a new model for intuitionistic fuzzy multi-attributes decision making. The proposed model deals with the degree of membership and degree of nonmembership separately. It is resistant to extreme data.

1. Introduction

Since the theory of fuzzy sets was introduced by Zadeh (1965), there have been a number of studies carried out both in the theoretical as well as practical areas. An important generalization of the fuzzy sets in the sense of Zadeh are intuitionistic fuzzy sets(IFS) introduced by Atanassov (1986). Unlike "classical" fuzzy set, an IFS A does not require that the sum of the degrees of membership and nonmembership of an element to A equals one. In other words, an IFS allows for a degree of hesitation. Notice that the IFS was reintroduced by Gau and Buehrer (1993) under the name of vague sets. Bustince and Burillo (1996) pointed out that vague sets are in fact IFSs.

Fuzzy sets theory has been applied successfully to many fields including multi-attribute decision making (MADM) (Barrenechea, Fernandez, Pagola, Chiclana, & Bustince, 2014; Chakrabortty, Pal, & Nayak, 2013; Dubois & Prade, 1980; Huang, Zhuang, & Li, 2013; Pedrycz, 2014; Xu & Da, 2002; Yager, 1988). In some real-world situations, decision makers are faced with a facet of hesitation and as a result some evaluations can be conveniently realized with the use of IFSs. For example, in a voting process, we witness approvals, rejections and abstentions. To deal with these situations, some authors employed similar methods used in fuzzy MADM problems. For example, some well-known aggregation operators, such as the weighted arithmetic (geometric) average operators and the ordered weighted arithmetic (geometric) average operators, have been generalized to the intuitionistic fuzzy environment (Xu, 2007; Xu & Yager, 2006; Yang & Chen, 2012). However these operators exhibit a common drawback: they are easily affected by some extreme data.

The main objective of this paper is to develop a new model for MADM problems under intuitionistic fuzzy environment, which is not easily affected by extreme data. The organization of the paper is as follows. After summarization of some previous results about IFSs (in Section 2), in Section 3, we discuss the linear orders for IFVs. It introduces a linear order—we use Szmidt and Kacprzyk's (Szmidt & Kacprzyk) order to distinguish two IFVs whenever Atanassov's order does not work. It is proved in this section that the introduced order is equivalent to the lexicographical one. Section 4 introduces a weighted arithmetic average operators, and then develops a new model for intuitionistic fuzzy MADM problems. We apply the proposed model to a real-life problem in Section 5. Section Section 6 delivers some conclusions.

2. Preliminaries

In this section, we briefly recall some essential ideas concerning intuitionistic fuzzy sets.

Definition 2.1 (Atanassov, 1999, 2012)**.** Let X be a universal set. An IFS A on U can be mathematically expressed as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in X\}$, where the maps $\mu_A\colon X \to [0, 1]$ and $\nu_A\colon X \to [0, 1]$ define the membership and nonmembership degrees of an element $x \in X$ to A, respectively, and such that they satisfy the following relationship $0 \le \mu_A(x) + \nu_A(x) \le 1$ for any $x \in X$.

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The function $\pi : X \to [0, 1]$ defined by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is referred to as the degree of hesitation with the membership of an element $x \in X$ to A. Obviously, if $\pi_A(x) = 0$ for each $x \in X$ then the IFS A becomes a fuzzy set in the sense of Zadeh (1965).

Atanassov (1986), De, Biswas, and Roy (2000) introduced the basic operations on IFSs.

Definition 2.2. Let A and B be IFSs.

(1) (Complement) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle, x \in X\}$;
(2) (Intersection) $A \bigcap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle, x \in X\}$;
(3) (Union) $A \bigcup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle, x \in X\}$;
(4) (Sum) $A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle, x \in X\}$;
(5) (Product) $A \cdot B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle, x \in X\}$;
(6) (Scale Multiplication) $nA = \{\langle x, 1 - (1 - \mu_A(x))^n, (\nu_A(x))^n\rangle, x \in X\}$;
(7) (Power) $A^n = \{\langle x, (\mu_A(x))^n, 1 - (1 - \nu_A(x))^n\rangle, x \in X\}$.

Atanassov employed the commonly used t-norm $T_P(x, y) = xy$ (product) and its dual t-conorm $S_P(x, y) = x + y - xy$ (probabilistic sum) to define the sum and the product of two IFSs. Xu and Yager (2006) and Xu (2007) proposed the concept of IFVs.

Definition 2.3. An IFV α is defined as an ordered pair $\langle u_\alpha, \nu_\alpha\rangle$ satisfying $u_\alpha, \nu_\alpha \geq 0$ and $u_\alpha + \nu_\alpha \leq 1$.

In this paper, the set of all IFVs will be denoted as Θ.

Based on the operations of IFSs in Definition 2.2, Xu and Yager defined a weighted arithmetic (geometric) average of IFVs.

Definition 2.4 (Xu, 2007; Xu & Yager, 2006)**.** Let $\{\alpha_i\}_{i=1}^n \subset \Theta$ and $\omega = (\omega_1, \ldots, \omega_n)^T$ be a weight vector.

(1) The intuitionistic fuzzy weighted arithmetic average operator IFWA is defined by

$$IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \Sigma_{i=1}^n \omega_i \alpha_i.$$

(2) The intuitionistic fuzzy weighted geometric average operator IFWG is defined by

$$IFWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \prod_{i=1}^n \alpha_i^{\omega_i}.$$

Suppose that $\alpha_i = \langle u_i, \nu_i\rangle$. One can easily prove the following two equalities:

$$IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle 1 - \prod_{i=1}^n (1 - u_i)^{\omega_i}, \prod_{i=1}^n \nu_i^{\omega_i} \right\rangle;$$

$$IFWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle \prod_{i=1}^n u_i^{\omega_i}, 1 - \prod_{i=1}^n (1 - \nu_i)^{\omega_i} \right\rangle.$$

The IFWA and IFWG operators have been widely applied to MADM problems (Li, 2011; Yang & Chen, 2012). These two operators are dual in the following sense.

Proposition 2.5. *Let $\alpha_i = \langle u_i, \nu_i\rangle, i = 1, 2, \ldots, n$ be IFVs and $\omega = (\omega_1, \ldots, \omega_n)^T$ be a weight vector. Then we have that*

$$IFWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overline{IFWA_\omega(\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_n)}.$$

The following example shows that in some extreme cases, if we use the IFWA operator then we may obtain a result that is totally different from the one we produce when using the IFWG operator, see also Beliakov, Bustince, Goswami, Mukherjee, and Pal (2011).

Example 2.6. Let $\{A_1, \ldots, A_m\}$ be the set of alternatives and $X = \{x_1, \ldots, x_n\}$ the attributes set. Suppose A_1 is described by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ satisfying $\alpha_1 = \langle 0, 1\rangle, \alpha_2 = \langle 1, 0\rangle$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is an arbitrary weight vector with $\omega_1 > 0$, $\omega_2 > 0$. If we use IFWA operator then $IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \langle 1, 0\rangle$, which implies that A_1 is the best choice (no matter what the other values are). But if we use IFWG operator then $IFWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \langle 0, 1\rangle$, which implies that A_1 is the worst choice.

From the above observations, we conclude that the IFWA operator is easily affected by extremely large values of data, while the IFWG operator is easily affected by extremely low data. To overcome this shortage, Beliakov et al. (2011) (see also Xia, Xu, & Zhu (2012)) introduced averaging operators for IFVs by using a continuous Archimedean t-norm and its dual t-conorm. We have

$$IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle h^{-1}\left(\sum_{i=1}^n w_i h(u_i)\right), g^{-1}\left(\sum_{i=1}^n w_i g(\nu_i)\right)\right\rangle, \tag{2.1}$$

where g is the additive generator of a continuous Archimedean t-norm and h is the additive generator of its dual t-conorm. Beliakov et al. (2011) also pointed out that (2.1) is consistent with the operation on ordinary fuzzy sets if and only if the t-norm is Łukasiewicz one. In this case, (2.1) becomes

$$IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle \sum_{i=1}^n w_i u_i, \sum_{i=1}^n w_i \nu_i \right\rangle.$$

Notice that this operator was discussed in Xu and Yager (2009), and also introduced in Chen and Tan (1994). This operator can be obtained by computing the weighted arithmetic means of the membership degree and the nonmembership degree, respectively. Similarly, we can introduce the following operator (although it is sensitive for extreme data), which we will call it the intuitionistic fuzzy pseudo weighted geometric average operator (IFPWG), by computing the weighted geometric means of the membership degree and the nonmembership degree, respectively. That is, we have

$$IFPWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle \sum_{i=1}^n u_i^{w_i}, \sum_{i=1}^n \nu_i^{w_i} \right\rangle.$$

It is easy to see that IFPWG is monotonic with respect to the partial order introduced by Atanassov (1999), refer to Section 3. That is, if $\alpha_i = \langle u_i, \nu_i\rangle, \alpha_i^\star = \langle u_i^\star, \nu_i^\star\rangle, i = 1, 2, \ldots, n$ be IFVs with both $u_i \leq u_i^\star$ and $\nu_i \geq \nu_i^\star$ for any i, then $IFPWG_\omega(\alpha_1, \ldots, \alpha_n) \leq IFPWG_\omega(\alpha_1^\star, \ldots, \alpha_n^\star)$ for any weight vector ω. In virtue of the idempotency of the weighted geometric average, we conclude that $IFPWG_\omega(\alpha, \alpha, \ldots, \alpha) = \alpha$ for any IFV α. Moreover, from the monotonicity and the idempotency of the IFPWG operator, we know that IFPWG is bounded, that is, $\underline{\alpha} \leq IFPWG_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \bar{\alpha}$, where $\underline{\alpha} = \cap_{i=1}^n \alpha_i = \langle \min_i\{u_i\}, \max_i\{\nu_i\}\rangle$ and $\bar{\alpha} = \cup_{i=1}^n \alpha_i = \langle \max_i\{u_i\}, \min_i\{\nu_i\}\rangle$. We also point out that IFPWG cannot be deduced from (2.1).

3. Linear orders of IFVs

One of the crucial problems in the context of decision making (when the IFVs are used) is the ranking of IFVs. Atanassov (1999) proposed an order for IFVs:

Let $\alpha = \langle u_1, \nu_1\rangle, \beta = \langle u_2, \nu_2\rangle$ be two IFVs. We say that $\alpha \leq_A \beta$ if both $u_1 \leq u_2$ and $\nu_1 \geq \nu_2$.

Unfortunately, $\leq_A$ is just a partial order. In order to apply IFVs to decision making, a linear(total) order is desired. There are several total orders of IFVs available in the literature (see Liu & Wang, 2007, for example). Here we recall the orders proposed by Bustince, Fernandez, Kolesárová, and Mesiar (2013a), Chen and Tan (1994), Hong and Choi (2000) and Szmidt and Kacprzyk (2009).