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### SHORT COMMUNICATION

## Entropy generation analysis of magneto hydrodynamic flow of a nanofluid over a stretching sheet



### M. Govindaraju<sup>a</sup>, N. Vishnu Ganesh<sup>a</sup>, B. Ganga<sup>b</sup>, A.K. Abdul Hakeem<sup>a,\*</sup>

<sup>a</sup> Department of Mathematics, Sri Ramakrishna Mission Vidhyalaya College of Arts and Science, Coimbatore 641 020, India <sup>b</sup> Department of Mathematics, Providence College for Women, Coonoor 643 104, India

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#### **KEYWORDS**

Nanofluid; Entropy generation; Similarity transformation; Magnetic field; Stretching sheet **Abstract** An analysis is carried out to study the entropy generation of an incompressible, MHD flow of water based nanofluid over a stretching sheet. The analytical solutions of the governing non-dimensional nonlinear ordinary differential equations are presented in terms of hypergeometric functions and used to compute the entropy generation number. The effects of the physical parameters on velocity and temperature profiles are already studied in our previous work [13]. This work is extended to discuss the effects of magnetic parameter, nanoparticle volume fraction, Hartmann number and the dimensionless group parameter on the entropy generation for Cu, Ag,  $Al_2O_3$  and  $TiO_2$  nanoparticles. The local skin friction coefficient and reduced Nusselt number are tabulated.

MATHEMATICS SUBJECT CLASSIFICATION: 76D10; 76W05; 80A20; 80A99; 82D80

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#### 1. Introduction

Entropy of a thermo dynamical system refers to the unavailability of useful work. Physically entropy generation is associated with thermo dynamical irreversibility, which is a common phenomenon in all kinds of heat transfer designs. Greater rate of entropy generation in any thermal system destroys the

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useful work and greatly reduces the efficiency of the system. Bejan [1,2] presented a method named Entropy Generation Minimization (EGM) to measure and optimizes the disorder or disorganization generated during a process specifically in the fields of refrigeration (cryogenics), heat transfer, storage and solar thermal power conversion. The entropy generation analysis of nanofluids investigated by several authors in different geometries [3–9].

The boundary layer flow over a continuously stretching surface finds many important applications in engineering processes, such as polymer extrusion and drawing of plastic films, and the applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. The effect of magnetic field on nanofluids studied by the followers

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<sup>\*</sup> Corresponding author. Tel.: +91 9442401998.

E-mail address: abdulhakeem6@gmail.com (A.K. Abdul Hakeem). Peer review under responsibility of Egyptian Mathematical Society.

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| $B_0$               | magnetic field strength                  | $k_f$         | thermal conductivity of the base fluid       |
|---------------------|--|---------------|--|
| Br                  | Brinkman number                          | $k_s$         | thermal conductivity of the nanoparticles    |
| Ha                  | Hartmann number                          | $\sigma$      | electric conductivity                        |
| M                   | Kummer's function                        | $\Omega$      | dimensionless temperature difference         |
| Mn                  | magnetic parameter                       | $\phi$        | the solid volume fraction                    |
| $N_s$               | entropy generation number                | $ ho_{nf}$    | the effective density of the nanofluid       |
| Pr                  | Prandtl number                           | $\rho_f$      | density of the pure fluid                    |
| $Re_{x}^{1/2}C_{f}$ | local skin friction coefficient          | $\rho_s$      | density of the nanoparticles                 |
| $Re_x^{-1/2}Nu_x$   | reduced Nusselt number                   | $\mu_{nf}$    | effective dynamic viscosity of the nanofluid |
| $S_{G}$             | local volumetric entropy generation rate | $\mu_f$       | dynamic viscosity of the basic fluid         |
| $(S_G)_0$           | characteristic entropy generation rate   | η             | space variable                               |
| T                   | local temperature of the fluid           | $\alpha_{nf}$ | thermal diffusivity of the nanofluid         |
| $k_{nf}$            | thermal conductivity of the nanofluid    | 5             | -  |

[10–12]. Very recently, we investigated the effect of magnetic field on water based nanofluid over a stretching sheet numerically [13] and also we studied the MHD flow of nanofluid with thermal radiation effect both analytically and numerically [14].

The purpose of this attempt is to analyse the entropy generation of magneto hydrodynamic flow of an incompressible viscous nanofluid over a stretching sheet analytically. The analytical solutions of dimensionless governing equations are presented in terms of hypergeometric function. The entropy generation is calculated using the entropy relation by substituting the velocity and temperature fields obtained from the momentum and entropy equations.

#### 2. Formulation of the problem

The entropy analysis for a steady laminar two-dimensional flow of an incompressible viscous nanofluid past a linearly semi-infinite stretching sheet is studied with magnetic field effect. We also consider influence of a constant magnetic field of strength  $B_0$  which is applied normally to the sheet. The temperature at the stretching surface takes the constant value  $T_w$ , while the ambient value, attained as y tends to infinity, takes the constant value  $T_{\infty}$ . It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field (as the magnetic Reynolds number is small). The fluid is a water based nanofluid containing different types of nanoparticles: Copper (Cu), Aluminium (Al<sub>2</sub>O<sub>3</sub>), Silver (Ag) and Titanium Oxide (TiO<sub>2</sub>). It is also assumed that the base fluid water and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are considered as in [13]. Under the above assumptions, the governing equations can be written in non-dimensional (see [13]) form as

$$F''' + (1 - \phi)^{2.5} \{ [1 - \phi + \phi(\rho_s/\rho_f)] (FF'' - F'^2) - Mn F' \} = 0$$
(1)

$$\theta'' + \frac{Prk_f[1 - \phi + \phi(\rho C_p)_s/(\rho C_p)_f]}{k_{nf}}F\theta' = 0$$
<sup>(2)</sup>

with the corresponding boundary conditions

$$F = 0 \quad F' = 1 \quad \text{at} \quad \eta = 0,$$
  

$$F' \to 0 \quad \text{as} \quad \eta \to \infty$$
(3)

$$\theta(0) = 1$$
 and  $\theta(\infty) = 0$  (4)

where  $\phi$  is the solid volume fraction,  $\rho_f$  and  $\rho_s$  are the densities of the base fluid and nanoparticles,  $(\rho C_p)_f$  and  $(\rho C_p)_s$  are the specific heat parameters of the base fluid and nanoparticles,  $k_f$  is the thermal conductivity of the base fluid,  $k_{nf}$  the thermal conductivity of the nanofluid, Pr is the Prandtl number and *Mn* is the magnetic parameter.

#### 3. Analytical solutions of the flow field and the heat transfer

The exact solution to differential Eq. (1) satisfying the boundary condition (3) is obtained as (see Anjali Devi and Ganga [15])

$$F(\eta) = \frac{1 - e^{-m\eta}}{m} \tag{5}$$

where m is the parameter associated with the nanoparticle volume fraction, the magnetic field parameter, the fluid density and the nanoparticle density as follow

$$m = \sqrt{(1-\phi)^{2.5} [Mn + 1 - \phi + \phi(\rho_s/\rho_f)]}$$
(6)

The analytical solution of (2) satisfying (4) interms of  $\eta$  is obtained as

$$\theta(\eta) = e^{-ma_0\eta} \frac{M[a_0, a_0 + 1, -a_0 e^{-m\eta}]}{M[a_0, a_0 + 1, -a_0]}$$
(7)

where *M* is the Kummer's function ([15]),  $\alpha = \frac{k_{nf}}{k_{f}(1-\phi+\phi\frac{(\rho C_{P})_{s}}{(\rho C_{P})_{s}})}$  and  $a_0 = \frac{Pr}{am^2}$ .

The skin friction can be written as

$$Re_x^{1/2}C_f = -\frac{2F''(0)}{(1-\phi)^{2.5}}$$
  
=  $-\frac{2}{(1-\phi)^{2.5}}\sqrt{(1-\phi)^{2.5}[Mn+1-\phi+\phi(\rho_s/\rho_f)]}$  (8)

The non-dimensional wall temperature gradient derived from Eq. (7) reads as

Nomenclature

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