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Original Article

The entire sequence over Musielak *p*-metric space

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Abstract In this paper, we introduce fibonacci numbers of $\Gamma^2(F)$ sequence space over *p*-metric spaces defined by Musielak function and examine some topological properties of the resulting these spaces.

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1. Introduction

Throughout w, Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences, respectively.

We write w^2 for the set of all complex sequences (x_{mn}), where $m, n \in \mathbb{N}$, the set of positive integers. Then, w^2 is a linear space under the coordinate wise addition and scalar multiplication.

Some initial works on double sequence spaces are found in Bromwich [1]. Later on it was investigated by Hardy [2], Moricz [3], Moricz and Rhoades [4], Basarir and Solankan [5], Tripathy et al. [6–18], Turkmenoglu [19], Raj [20–26] and many others.

We procure the following sets of double sequences:

$$\mathcal{M}_u(t) := \left\{(x_{mn}) \in w^2 : sup_{m,n\in N}|x_{mn}|^{t_{mn}} < \infty\right\},$$

$$\mathcal{C}_p(t) := \left\{(x_{mn}) \in w^2 : p-\lim_{m,n\to\infty}|x_{mn}-|^{t_{mn}} = 1 \ for\ some \in \mathbb{C}\right\},$$

$$\mathcal{C}_{0p}(t) := \left\{(x_{mn}) \in w^2 : p-\lim_{m,n\to\infty}|x_{mn}|^{t_{mn}} = 1\right\},$$

$$\mathcal{L}_u(t) := \left\{(x_{mn}) \in w^2 : \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|x_{mn}|^{t_{mn}} < \infty\right\},$$

$$\mathcal{C}_{bp}(t) := \mathcal{C}_p(t)\bigcap\mathcal{M}_u(t) \text{ and } \mathcal{C}_{0bp}(t) = \mathcal{C}_{0p}(t)\bigcap\mathcal{M}_u(t);$$

where $t = (t_{mn})$ be the sequence of strictly positive reals t_{mn} for all $m, n \in \mathbb{N}$ and $p - \lim_{m,n\to\infty}$ denotes the limit in the Pringsheim's sense. In the case $t_{mn} = 1$ for all $m, n \in \mathbb{N}$; $\mathcal{M}_u(t), \mathcal{C}_p(t), \mathcal{C}_{0p}(t), \mathcal{L}_u(t), \mathcal{C}_{bp}(t)$ and $\mathcal{C}_{0bp}(t)$ reduce to the sets $\mathcal{M}_u, \mathcal{C}_p, \mathcal{C}_{0p}, \mathcal{L}_u, \mathcal{C}_{bp}$ and $\mathcal{C}_{0bp}$, respectively. Now, we may summarize the knowledge given in some document related to the double sequence spaces. Gökhan et al. [27,28] have proved that $\mathcal{M}_u(t)$ and $\mathcal{C}_p(t), \mathcal{C}_{bp}(t)$ are complete paranormed spaces of double sequences and obtained the α-, β-, γ-duals of the spaces $\mathcal{M}_u(t)$ and $\mathcal{C}_{bp}(t)$. Quite recently, in her PhD thesis, Zeltser [29] has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen et al. [30–35] have independently introduced the statistical convergence and Cauchy

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for double sequences and established the relation between statistical convergent and strongly Cesàro summable double sequences. Altay and Başar [36] have defined the spaces $\mathcal{BS}, \mathcal{BS}(t), \mathcal{CS}_p, \mathcal{CS}_{bp}, \mathcal{CS}_r$ and $\mathcal{BV}$ of double sequences consisting of all double series whose sequence of partial sums are in the spaces $\mathcal{M}_u, \mathcal{M}_u(t), \mathcal{C}_p, \mathcal{C}_{bp}, \mathcal{C}_r$ and $\mathcal{L}_u$, respectively, and also examined some properties of those sequence spaces and determined the α-duals of the spaces $\mathcal{BS}, \mathcal{BV}, \mathcal{CS}_{bp}$ and the $\beta(\vartheta)$-duals of the spaces $\mathcal{CS}_{bp}$ and $\mathcal{CS}_r$ of double series. Başar and Sever [37] have introduced the Banach space$\mathcal{L}_q$ of double sequences corresponding to the well-known space ℓ_q of single sequences and examined some properties of the space $\mathcal{L}_q$. Recently Subramanian and Misra [38] have studied the space $\chi^2_M(p, q, u)$ of double sequences and proved some inclusion relations.

The class of sequences which are strongly Cesàro summable with respect to a modulus was introduced by Maddox [39] as an extension of the definition of strongly Cesàro summable sequences. Cannor [40] further extended this definition to a definition of strong A-summability with respect to a modulus where $A = (a_{n,k})$ is a nonnegative regular matrix and established some connections between strong A-summability, strong A-summability with respect to a modulus, and A-statistical convergence. In Pringsheim [41] the four dimensional matrix transformation $(Ax)_{k,\ell} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{k\ell}^{mn} x_{mn}$ was studied extensively by Robison and Hamilton.

We need the following inequality in the sequel of the paper. For $a, b, \geq 0$ and $0 < p < 1$, we have

$$(a+b)^p \leq a^p + b^p. \tag{1.1}$$

The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (s_{mn}) is convergent, where $s_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m, n \in \mathbb{N})$.

A sequence $x = (x_{mn})$ is said to be double analytic if $sup_{mn}|x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called double gai sequence if $|x_{mn}|^{1/m+n} \to 0$ as $m, n \to \infty$. The double gai sequences will be denoted by Γ^2. Let $\phi = \{all\ finite\ sequences\}$.

Consider a double sequence $x = (x_{ij})$. The $(m, n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\Im_{ij}$ for all $m, n \in \mathbb{N}$; where $\Im_{ij}$ denotes the double sequence whose only nonzero term is a $\frac{1}{(i+j)!}$ in the $(i, j)^{th}$ place for each $i, j \in \mathbb{N}$.

An FK-space(or a metric space)X is said to have AK property if $(\Im_{mn})$ is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$.

An FDK-space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \to (x_{mn})(m, n \in \mathbb{N})$ are also continuous.

Let M and Φ be mutually complementary modulus functions. Then, we have

(i) For all $u, y \geq 0$,

$$uy \leq M(u) + \Phi(y), (Young'sinequality) \tag{1.2}$$

[*See Kampthan et al.*, [42]].

(ii) For all $u \geq 0$,

$$u\eta(u) = M(u) + \Phi(\eta(u)). \tag{1.3}$$

(iii) For all $u \geq 0$, and $0 < \lambda < 1$,

$$M(\lambda u) \leq \lambda M(u). \tag{1.4}$$

Lindenstrauss and Tzafriri [43] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty,\ for\ some\ \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$\|x\| = inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p (1 \leq p < \infty)$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

A sequence $f = (f_{mn})$ of modulus function is called a Musielak-modulus function. A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = sup\{|v|u - f_{mn}(u) : u \geq 0\}, m, n = 1, 2, \ldots$$

is called the complementary function of a Musielak-modulus function f. For a given Musielak modulus function f, the Musielak-modulus sequence space t_f is defined by

$$t_f = \left\{ x \in w^2 : I_f(|x_{mn}|)^{1/m+n} \to 0 as m, n \to \infty \right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}(|x_{mn}|)^{1/m+n}, x = (x_{mn}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x, y) = sup_{mn}\left\{ inf\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}\left(\frac{|x_{mn}|^{1/m+n}}{mn}\right)\right) \leq 1 \right\}.$$

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [44] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\},$$

for $Z = c, c_0$ and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$.

Here c, c_0 and ℓ_∞ denote the classes of convergent, null and bounded scalar valued single sequences respectively. The difference sequence space bv_p of the classical space ℓ_p is introduced and studied in the case $1 \leq p \leq \infty$ by Başar and Altay and in the case $0 < p < 1$ by Altay et al. The spaces $c(\Delta), c_0(\Delta), \ell_\infty(\Delta)$ and bv_p are Banach spaces normed by

$$\|x\| = |x_1| + sup_{k\geq 1}|\Delta x_k| \text{ and } \|x\|_{bv_p} = \left(\sum_{k=1}^{\infty}|x_k|^p\right)^{1/p}, (1 \leq p < \infty).$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta) = \left\{ x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z \right\},$$

where $Z = \Lambda^2, \chi^2$ and $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m, n \in \mathbb{N}$. The generalized difference double no-

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