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Original Article

# New types of continuity and openness in fuzzifying bitopological spaces

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**Abstract** In this paper, we introduce and study the concepts of semicontinuous mappings, $\alpha$–continuous mappings, semiopen mappings and $\alpha$–open mappings in fuzzifying bitopological spaces. The characterizations of these mappings along with their relationship with certain other mappings are investigated.

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## 1. Introduction

In 1965 [1], Zadeh introduced the fundamental concept of fuzzy sets. Since Chang introduced fuzzy sets theory into topology in 1968 [2], Wong, Lowen, Hutton, Pu and Liu, etc., discussed respectively various aspects of fuzzy topology [3–6].

In 1991–1993 [7–9], Ying introduced the concept of the fuzzifying topology with the sematic method of continuous valued logic. In 1994 [10], Park and Lee introduced and discussed the concepts of fuzzy semi-preopen sets and fuzzy semi-precontinuous mappings. Also, in 1994 Kumar [11,12] studied the concepts of fuzzy pairwise $\alpha$-continuity and pairwise pre-continuity and studied the concepts of semiopen sets, semi continuity and semiopen mappings in fuzzy bitopological spaces. In 1999 Khedr et al. [13], introduced the concepts of semi-open sets and semi-continuity in fuzzifying topology.

The study of bitopological spaces was first initiated by Kelley [14] in 1963. In 2003 Zhang and Liu [15], studied the concept of fuzzy $\theta_{(i,j)}$-closed, $\theta_{(i,j)}$-open sets in fuzzifying bitopological spaces. Also in [16], Gowrisankar et al. studied the concepts of $(i,j)$-pre open sets in fuzzifying bitopological spaces.

The structure of this paper is organized as follows: In Section (3) we study fuzzy continuity, open mapping in fuzzifying bitopological spaces and we introduce some results. In Section (4) we study $\alpha$-open set in fuzzifying bitopological spaces and we introduce the relationship between this set and preopen (resp. semiopen) sets. In Section (5) we define the concepts of semicontinuity, $\alpha$-semicontinuity in fuzzifying bitopological spaces and study the relationship between them. In Section (6) we define the concepts of fuzzy semiopen mapping,

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$\alpha$-open mapping, preopen mapping in fuzzifying bitopological spaces and study the relationship between them.

## 2. Preliminaries

Firstly, we display the fuzzy logical and corresponding set-theoretical notations used in this paper:

(1) A formula $\varphi$ is valid, we write $\models \varphi$ if and only if $[\varphi] = 1$ for every interpretation.
(2) $[\neg\alpha] = 1 - [\alpha]$,$[\alpha \wedge \beta] = \min([\alpha],[\beta])$,$[\alpha \rightarrow \beta] = \min(1,1 - [\alpha] + [\beta])$,
$[\forall x \alpha(x)] = \inf_{x \in X}[\alpha(x)]$, where $X$ is the universe of discourse.
(3) $[\alpha \vee \beta] := [\neg(\neg\alpha \wedge \neg\beta)]$; $[\alpha \leftrightarrow \beta] := [(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)]$;$[\exists x \alpha(x)] := [\neg(\forall x \neg\alpha(x))]$;-
$[\tilde{A} \subseteq \tilde{B}] := [\forall x(x \in \tilde{A} \rightarrow x \in \tilde{B})] = \inf_{x \in X}\min(1,1 - \tilde{A}(x) + \tilde{B}(x))$;$[A \equiv B] := [(\tilde{A} \subseteq \tilde{B}) \wedge (\tilde{B} \subseteq \tilde{A})]$, where $\tilde{A}$, $\tilde{B} \in \Im(X)$ and $\Im(X)$ is the family of all fuzzy sets in $X$.
(4) $[\alpha \dot{\wedge} \beta] := [\neg(\alpha \rightarrow \neg\beta)] = \max(0,[\alpha] + [\beta] - 1)$;
$[\alpha \dot{\vee} \beta] := [\neg\alpha \rightarrow \beta] = \min(1,[\alpha] + [\beta])$.

Secondly, we give the following definitions which are used in the sequel.

**Definition 2.1** [7]. Let $X$ be a universe of discourse, $P(X)$ is the family of subsets of $X$ and $\tau \in \Im(P(X))$ satisfy the following conditions:

(1) $\tau(X) = 1$ and $\tau(\phi) = 1$;
(2) for any $A,B,\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$;
(3) for any $\{A_\lambda : \lambda \in \Lambda\},\tau(\bigcup_{\lambda \in \Lambda} A_\lambda) \geq \bigwedge_{\lambda \in \Lambda} \tau(A_\lambda)$.

Then $\tau$ is a fuzzifying topological space.

**Definition 2.2** [7]. Let $(X,\tau)$ be a fuzzifying topological space.

(1) The family of all fuzzifying closed sets is denoted by $F \in \Im(P(X))$, and defined as follows: $A \in F := X \sim A \in \tau$, where $X \sim A$ is the complement of $A$.
(2) The neighborhood system of $x \in X$ is denoted by $N_x \in \Im(P(X))$ and defined as follows:
$N_x(A) = \sup_{x \in B \subseteq A} \tau(B)$.
(3) The closure $cl(A)$ of $A \subseteq X$ is defined as follows: $cl(A)(x) = 1 - N_x(X \sim A)$.
(4) The interior of $A \subseteq X$ is denoted by $int(A) \in \Im(P(X))$ and defined as follows: $int(A) = N_x(A)$.

**Definition 2.3** [9]. Let $(X,\tau)$ and $(Y,\sigma)$ be two fuzzifying topological spaces.

(1) A unary fuzzy predicate $C \in \Im(Y^X)$, called fuzzy continuity, is given as follows:
$f \in C := \forall u(u \in \sigma \rightarrow f^{-1}(u) \in \tau)$. i.e.,

$$C(f) = \inf_{u \in P(Y)} \min(1,1 - \sigma(u) + \tau(f^{-1}(u))).$$

(2) A unary fuzzy predicate $O \in \Im(Y^X)$, called fuzzy openness, is given as follows:
$f \in O := \forall u(u \in \tau \rightarrow f(u) \in \sigma)$. i.e.,

$$O(f) = \inf_{u \in P(X)} \min(1,1 - \tau(u) + \sigma(f(u))).$$

**Definition 2.4** [15]. Let $(X,\tau_1)$ and $(X,\tau_2)$ be two fuzzifying topological spaces. Then a system $(X,\tau_1,\tau_2)$ consisting of a universe of discourse $X$ with two fuzzifying topologies $\tau_1$ and $\tau_2$ on $X$ is called a fuzzifying bitopological space.

**Definition 2.5** [17]. Let $(X,\tau_1,\tau_2)$ be a fuzzifying bitopological space.

(1) The family of all fuzzifying $(i,j)$-semiopen sets, denoted by $s\tau_{(i,j)} \in \Im(P(X))$, is defined as follows:
$A \in s\tau_{(i,j)} := \forall x(x \in A \rightarrow x \in cl_j(int_i(A)))$, i.e.,
$s\tau_{(i,j)}(A) = \inf_{x \in A} cl_j(int_i(A))(x)$.
(2) The family of all fuzzifying $(i,j)$-semiclosed sets, denoted by $sF_{(i,j)} \in \Im(P(X))$, is defined as follows:
$A \in sF_{(i,j)} := X \sim A \in s\tau_{(i,j)}$.

**Definition 2.6** [17]. Let $(X,\tau_1,\tau_2)$ be a fuzzifying bitopological space and $x \in X$.

(1) The $(i,j)$-semi neighborhood system of $x$ is denoted by $sN_x^{(i,j)} \in \Im(P(X))$ and defined as
$A \in sN_x^{(i,j)} := \exists B(B \in s\tau_{(i,j)} \wedge x \in B \subseteq A)$, i.e.,
$sN_x^{(i,j)}(A) = \sup_{x \in B \subseteq A} s\tau_{(i,j)}(B)$.
(2) The $(i,j)$-semi derived set $sd_{(i,j)}(A)$ of $A$ is defined as follows:
$x \in sd_{(i,j)}(A) := \forall B(B \in sN_x^{(i,j)} \rightarrow B \cap (A \sim \{x\}) \neq \phi)$,
i.e., $sd_{(i,j)}(A)(x) = \inf_{B \cap (A \sim \{x\}) = \phi}(1 - sN_x^{(i,j)}(B))$.
(3) The Fuzzifying $(i,j)$-semi closure of $A \subseteq X$, is denoted by $scl_{(i,j)}(A)$ and defined as follows:
$x \in scl_{(i,j)}(A) := \forall B((B \supseteq A) \wedge (B \in sF_{(i,j)}) \rightarrow x \in B)$,
i.e., $scl_{(i,j)}(A)(x) = \inf_{x \notin B \supseteq A}(1 - sF_{(i,j)}(B))$.
(4) The $(i,j)$-semi interior of $A \subseteq X$ is defined as follows:
$sint_{(i,j)}(A)(x) = sN_x^{(i,j)}(A)$.
(5) The $(i,j)$-semi exterior of $A \subseteq X$ is defined as follows:
$x \in sext_{(i,j)}(A) := x \in sint_{(i,j)}(X \sim A)$, i.e.,
$sext_{(i,j)}(A)(x) = sint_{(i,j)}(X \sim A)(x)$.
(6) The $(i,j)$-semi boundary of $A \subseteq X$ is defined as follows:
$x \in sb_{(i,j)}(A) := (x \notin sint_{(i,j)}(A)) \wedge (x \notin sint_{(i,j)}(X \sim A))$,
i.e., $sb_{(i,j)}(A)(x) = \min(1 - sint_{(i,j)}(A)(x),1 - sint_{(i,j)}(X \sim A)(x))$.

**Definition 2.7** [16]. Let $(X,\tau_1,\tau_2)$ be a fuzzifying bitopological space. The family of fuzzifying $(i,j)$-preopen sets, denoted by $p\tau_{(i,j)} \in \Im(P(X))$, is defined as follows:
$A \in p\tau_{(i,j)} := \forall x(x \in A \rightarrow x \in int_i(cl_j(A)))$. i.e.,
$p\tau_{(i,j)}(A) = \inf_{x \in A} int_i(cl_j(A))(x)$.

**Definition 2.8** [16]. Let $(X,\tau_1,\tau_2)$ and $(X,\sigma_1,\sigma_2)$ be two fuzzifying bitopological spaces. A unary fuzzy predicate $PC_{(i,j)} \in \Im(Y^X)$, called fuzzy precontinuity, is given as follows:
$PC_{(i,j)}(f) := \forall v(v \in \sigma_i \rightarrow f^{-1}(v) \in p\tau_{(i,j)})$. i.e.,

$$PC_{(i,j)}(f) = \inf_{v \in P(Y)} \min(1,1 - \sigma_i(v) + p\tau_{(i,j)}(f^{-1}(v))).$$