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Original Article

Analysis of finite buffer queue with state dependent service and correlated customer arrivals

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Abstract This paper deals with a finite capacity queue with workload dependent service. The arrival of a customer follows Markovian arrival process (MAP). MAP is very effective arrival process to model massage-flow in the modern telecommunication networks, as these messages are very bursty and correlated in nature. The service time, which depends on the queue length at service initiation epoch, is considered to be generally distributed. Queue length distribution at various epoch and key performance measures have been obtained. Finally, some numerical results have been discussed to illustrate the numerical compatibility of the analytic analysis of the queueing model under consideration.

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1. Introduction

In many practical queueing situations, a long queue of customers waiting for service is quite common which in turn causes poor system performance (which is measured in terms of loss/rejection/blocking probability). To avoid this inconvenient situation, the decision maker often decide to control arrival rates or service rates to reduce blocking probabilities or congestion. Various queueing models have been studied for overload control to prevent congestion in telecommunication networks, in particular, ATM (asynchronous transfer mode) networks (Jain [1]). Overload control by controlling the service rate is discussed earlier by Choi and Choi [2], Choi et al. [3], Sriram and Lucantoni [4], Banerjee and Gupta [5], Banerjee et al. [6], etc. In particular, Choi and Choi [2] analyzed a finite buffer queue, where customers arrive according to the Markov modulated Poisson process ($MMPP$), with queue-length-dependent service. They considered that if the queue length at the service initiation epoch is less than or equal to a threshold limit (say, L), the service time follows G_1 distribution, and if the queue length at the service initiation-epoch exceeds the threshold limit, the service time distribution of the customers is G_2. They obtained queue length distribution at departure- and arbitrary-epoch. They also obtained performance measures, viz. loss probability, mean waiting time, etc.

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In this paper, we consider a finite capacity queueing model where arrival follows MAP. $MMPP$, with matrix representation $(\mathbf{Q}, \mathbf{\Lambda})$, is a special case of MAP, with matrix representation $(\mathbf{C}, \mathbf{D})$. That is, if one consider $\mathbf{Q} = \mathbf{C} + \mathbf{D}$ and $\mathbf{\Lambda} = \mathbf{D}$, then the arrival process MAP will reduces to $MMPP$. Also in this paper the service time distribution of a customer is considered to be G_n, i.e., service time of a customer depends on the number of customers (n) present in the system at service initiation epoch of a customer and also generally distributed. Hence, for $G_n = G_1$ $(1 \le n \le L)$, $G_n = G_2$ $(n > L)$, $\mathbf{D} = \mathbf{\Lambda}$ and $\mathbf{Q} = \mathbf{C} + \mathbf{D}$ the model presented in this paper will be reduced to the model discussed by Choi and Choi [2]. Therefore, the model considered in this paper is more general and complex than the one considered by Choi and Choi [2]. One may note here that the results obtained in this paper cannot be deduced from [2], whereas the results obtained in [2] can be deduced from the present study as a special case.

The analysis of this paper is carried out as follows: first departure-epoch probabilities have been obtained by using the embedded Markov chain technique. Then using the supplementary variable technique and considering the supplementary variable as remaining service time of a customer, relations between arbitrary- and departure-epoch probabilities have been obtained. Distribution of the number of customers in the queue at arrival-epoch has been also obtained. Performance measures such as average number of customers in the queue, probability of blocking and average waiting time of a customer in the queue have been obtained. Then computational procedure when service time distribution is phase type (PH-distribution) has been discussed. It should be noted here that many distribution (viz. exponential, hyperexponential, hypoexponential, Erlang, Coxian, etc.) in continuous time set up can be approximated by PH-distribution. Finally, comparative studies of queue-length-dependent service with the one when service time of the customers are independent of the queue length have been carried out by using self explanatory graphs as the effect of buffer-size on performance measures. These comparative studies establish the fact that our model is more effective than the one when service time is independent of the queue length. For the sake of notational convenience we denote this model by $MAP/G_n/1/N$ for future reference. It may be remarked here that in a special case when $G_n = G$, i.e., service time of the customers is independent of the queue size, the model reduces to simple $MAP/G/1/N$ queue (Gupta and Laxmi [7]). Gupta and Laxmi [7] analyzed $MAP/G/1$ with finite/infinite buffer and obtained relations among the queue size distributions at departure-, arbitrary- and arrival-epoch using the supplementary variable technique. However, they did not provide the computational procedure and numerical illustrations. One can easily obtain numerical illustrations of $MAP/G/1/N$ queue from the present study.

ATM networks support diverse traffics with different service characteristics such as voice, data and video. In B-ISDN/ATM network, IP packets or cells of voice, video, data are sent over a common transmission channel on statistical multiplexing basis. These traffics are statistically multiplexed and transmitted in superhigh speed. Also, it is seen that the traffic in modern communication networks is highly irregular (e.g., bursty and correlated). A good representation of such traffic is a Markovian arrival process (MAP). Hence, the model discussed in this paper can be used to control congestion in the telecommunication networks by controlling the transmission rate (service rate) depending on the number of the packet waiting in the queue (queue length). In recent years there has been a growing interest to analyze queues with input process as MAP which is a rich class of point processes containing many familiar arrival processes, such as, Poisson process, interrupted Poisson process (IPP), PH-renewal process, Markov modulated Poisson process ($MMPP$), etc. Lucantoni et al. [8]. Later, queueing models with MAP have been studied extensively, in past, see e.g., [9–15] and many others. For recent development in MAP, see, [16–21] and the references therein.

A real life application of the proposed model may be observed in modern telecommunication networks, viz. advanced wireless and mobile internet networks. Demand for high speed wireless internet access, voice and multimedia applications results in the popularity of technologies like 3G and 4G. IEEE 802.16 is a telecommunication standard technology designed to support a wide variety of wireless and wired broadband access of multimedia applications with expectation to provide Quality of Service (QoS). The basic IEEE 802.16 system consists of one Base Station (BS) and one (or more) Subscriber Station (SS). BS acts as a transmitter to transfer all type of data (voice, video, data, etc.). Transmissions take place through two independent channels, downlink channel (from BS to SS) and uplink channel (from SS to BS). In case of internet traffic, downlink gets higher preference over the uplink. This internet traffic flow system may be modeled as a multimedia data transmission system over a wireless channel, where packets are queued at the transmitter (BS). Since the incoming traffic in IEEE 802.16 is irregular and bursty in nature, causing correlation in inter-arrival time, arrival process can be modeled using MAP. Due to rapid increase in the popularity of 2G and 3G system, it has become necessary to develop new schemes for congestion control to reduce queue length, waiting time and probability of rejection (blocking). In this direction if one consider that the BS transmits data, depending on the queue size, to a SS, the system can be modeled and analyzed as queue length dependent service queue with MAP, which has been done (analytically) in this paper. This scheme of transmission of packets (e.g., from BS to SS) enhances the overall efficiency of the system as well as improves the QoS. Fig. 1 illustrates the data transmission system discussed above. The rest of this paper is organized as follows. In Section 2, description of the model and its analysis at various epoch is given. The computational procedure when service time follows phase type distribution are spelled out in Section 5. System performance measures and numerical results are given in Sections 4 and 5, respectively. The paper ends with some concluding remark in Section 6.

2. Model description and solution

Consider a single server queue where customers arrive according to the Markovian arrival process (MAP) with matrix representation $(\mathbf{C}, \mathbf{D})$ of dimension m. The generator $\mathbf{Q}^*$, governing the continuous time Markov chain governing the arrival process, is then given by $\mathbf{Q}^* = \mathbf{C} + \mathbf{D}$. Let $\boldsymbol{\delta}$ denotes the stationary probability vector of the Markov processes with generator $\mathbf{Q}^*$. That is, $\boldsymbol{\delta}$ is the unique (positive) probability vectors satisfying $\boldsymbol{\delta}\mathbf{Q}^* = \mathbf{0}$, $\boldsymbol{\delta}\boldsymbol{e} = 1$. The fundamental arrival rate of the stationary MAP is given by $\lambda^* = \boldsymbol{\delta}\mathbf{D}\mathbf{e}$. Here $\mathbf{e}$ and $\mathbf{0}$ are the $m \times 1$ vectors of ones and zeros, respectively. For more detail on this topic, see, Lucantoni et al. [8].