ARTICLE IN PRESS
PTEC-12360; No of Pages 11
Powder Technology xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

A fully coupled hydro-mechanical material point method for saturated dense granular materials

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ARTICLE INFO

Article history:
Received 10 June 2016
Received in revised form 10 February 2017
Accepted 14 February 2017
Available online xxxx

Keywords:
Hydro-mechanical coupling
Granular materials
Material point method
Large deformation

ABSTRACT

The stability of a dense granular assembly can be greatly reduced by a pore pressure of the interstitial fluid, and the body may fail and transit from a solid-like state to a fluid-like state. This process involves two major problems: large deformation and hydro-mechanical coupling. In this work, a three-dimensional fully coupled hydro-mechanical model using material point method (MPM) is developed. Darcy's law, considering the inertial effect, is adopted to govern the motion of interstitial water, and the conservation of momentum of the mixture is used to govern the motion of the solid, i.e., granular materials. The spatial discretization schemes for these equations are derived using the generalized integration material point method (GIMP), and the proposed coupled MPM formulation is implemented in a three-dimensional numerical code. The developed model is first quantitatively validated by comparing the simulation results of temporal evolution of spatial distribution of hydraulic pressure in a one-dimensional oedometer test with the analytical results. An experiment is designed to observe the failure of a saturated sand pile, in which the partial-saturated region is avoided by increasing the hydraulic head at the input boundary, and the kinetic energy of water is dissipated by a filtering cloth. The failure process is simulated with the MPM code. It is found that the location of the shear band in the simulation agrees with the location of the sliding surface in the experiment. The temporal evolutions of the spatial distributions of hydraulic pressure and the solid velocity distribution at a specific time are given to provide insight into the mechanism of the failure process. This work would be helpful in understanding the initiation mechanism of debris flows induced by rainfall, and sand production in gas hydrate-bearing sediments due to increasing fluid content associated with hydrate dissociation.

1. Introduction

A large number of geophysical hazards, such as landslides and debris flows, occur every year in mountainous areas in China and are usually agitated by heavy rains and/or earthquakes [1]. In these hazards, gaps between the solid particles are filled with interstitial water and air; thus, the water content affects the strength of the granular material. In the fully saturated region, the hydraulic pressure reduces the effective stress of the solid skeleton; however, in the partially saturated region, the matrix suction strengthens the connections between particles. Moreover, seepage causes changes in the water content, leading to a more complex situation. Studies of failure in fully saturated, dense granular materials are rather complicated due to large deformation and hydro-mechanical coupling. Meanwhile, gas production from gas hydrate-bearing sediments attracted international interest because of its potential to meet the growing global energy demand and ensure energy security. During trials at the Canadian Mallik gas hydrate site in 2007 [2] and at the Eastern Nankai Trough in Japan in 2013 [3], excessive amounts of sand migrated into the wells, preventing further gas production, and the operations were prematurely terminated. Researchers have highlighted the importance of incorporating pore water pressure into formulations of solid effective stress to better predict and understand the mechanisms involved in gas hydrate and sand production. Therefore, it is necessary to develop a fully coupled hydro-mechanical model for geo-hazard prediction/prevention and gas production from gas hydrate-bearing sediments. Many numerical methods have been proposed to model the mechanical problems of two-phase systems. According to solid particle modelling approaches, these methods can be generally classified into discontinuous and continuous methods.

In discontinuous methods, the discrete element method (DEM) is widely used to treat the solid phase [4]. The resultant force on each particle is obtained by summing all the forces, such as the

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http://dx.doi.org/10.1016/j.powtec.2017.02.022
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inter-particle contact force, fluid drag force, gravitational force and so on. An integration method is employed to compute the change in the position and velocity of each particle during a certain time step using Newton's laws of motion. Then, the bulk behaviors of solid particles can be studied. The fluid phase can be described by the Navier-Stokes equation, leading to the coupling of direct numerical simulation (DNS) and DEM [5]. Moreover, for turbulent flow, coupling of the large eddy simulation (LES) and DEM has been developed to expand studies of two-phase flows with high Reynolds number [6]. If the water is governed by the probability density function of velocity, coupling of the lattice Boltzmann method (LBM) and DEM can exactly calculate the interaction force between solid and water [7,8]. These discontinuous methods are relatively accurate, but the simulations are rather time consuming.

Continuous methods focus on the macroscopic behaviors of two-phase systems. For example, the mixture of solids and fluids can be treated as a single phase, i.e., a Bingham type fluid [9,10]. The drawback is that the introduced variables sometimes have no clear physical meanings or are difficult to measure [11]. A variety of finite element methods (FEM) have been developed for solid-fluid coupled problems in past decades using the $u-w-p$, $u-U$ and $u-p$ form governing equations [12,13]. Here, u is the velocity of water, w is the relative velocity between water and solid, U is the velocity of solid, and p is the hydraulic pressure. The FEM, as a robust spatial discretization method used to analyze mechanical problems, also exhibits disadvantages in large-deformational problems due to mesh distortion [14,15]. Recently, a multi-scale modelling scheme was developed, in which the constitutive relationship is extracted from the represent volume element (RVE) using DEM. Although this solution provides an interesting scheme for investigating two-phase problems without a phenomenological constitutive model, it is difficult to use in large-deformation problems because the macro analysis is still based on the FEM [16].

Originating from the particle-in-cell (PIC) method, the material point method (MPM) is a mesh-free method, combining the Eulerian description and Lagrangian description [17,18]. The analyzed continuum is divided into several sub-continuums represented by material points, and the physical variables of the sub-continuums, such as mass, momentum and energy, are assigned to the corresponding material points. Then, these points are connected with a background grid using weighting functions and their gradients, and the motion equation of grid node is solved to evolve the continuum. In comparison with the smoothed particle hydrodynamics (SPH) [19], the boundary conditions can be directly applied to the nodes of the background mesh, which is more straightforward and efficient than SPH.

The MPM has been used for simulations of one phase landslides [20,21], and some studies have been performed to investigate solid-fluid coupled problems. Zhang et al. [22] proposed a coupled material point method to study the dynamic responses of saturated soil subjected to contacts/impacts based on the $u-p$ form of the governing equations. In their work, only one layer of material points was used, resulting in a disadvantage associated with studying the relative motion between the solid and fluid. Similarly, Higo et al. [23] developed a procedure based on the $u-p$ formulation to simulate the responses of both fully and partially saturated elasto-plastic soil. The solid phase was solved using the MPM, and the fluid phase was solved by the finite difference method (FDM) using a background mesh of the MPM. However, they did not explicitly address geometric changes in the soil body. Zhang et al. [24] adopted two sets of material points to represent the deformation of a solid skeleton and interstitial fluid in fully saturated soil based on the $u-U$ form of the governing equations. Because the same interpolation function was used for the solid and the fluid layers, the formulation assumes only a small deformation of the soil. Abe et al. [25] used the MPM to simulate the deformation of a river levee embankment after seepage failure. Geometric changes in the model after failure agreed well with experimental results. In most previous studies, the relative acceleration of water with respect to the solid skeleton was not considered. Hence, it may not be suitable for application in high-frequency problems encountered in rapid deformation settings and other scenarios [26]. Bandara and Soga [27] developed a fully coupled MPM formulation considering the relative acceleration between fluid and solid using two sets of material points. Although the formation is derived from the full-saturation assumption, the numerical result was validated by comparison with a seepage failure experiment containing a partial-saturation zone. Soga et al. [28] summarized the various large-deformation analysis methods, with particular emphasis on the MPM, suggesting that relative acceleration is significant in two-phase simulations.

Experiments regarding structural failure due to seepage are essential when validating the numerical models. Experiments conducted to clarify the failure processes of landslides triggered by rainfall [29,30], and dam-break induced by water seepage can be found in many publications [31–33]. In reality, geophysical hazards seldom occur without a partially saturated region, except for dam overtopping. Zech et al. [34] presented a full-saturation experiment, in which the sediment movement was triggered by high water. However, in their experiment, the kinetic energy of water does not dissipate, resulting in sediment movement mainly induced by flushing rather than seepage.

Based on the work by Bandara and Soga [27], this study re-derives the formulation of a fully coupled MPM for saturated, dense granular materials using generalized interpolation material point method (GIMP), which is more reasonable for spatial discretization [35]. In consideration of the front and rear boundary conditions, which play significant roles in flow regimes of granular material [36], the proposed coupled MPM formulation is implemented in a three-dimensional numerical code rather than a two-dimensional code. A fully saturated experiment of seepage failure is conducted to validate our model. This paper is organized as follows. Section 2 presents the main assumptions, basic theories and spatial discretization scheme using the MPM. Validation of the numerical model with the oedometer test is given in Section 3. Details of the experimental observations of a saturated sand pile and the simulation results are presented in Section 4. Finally, Section 5 concludes the paper.

2. MPM for full-saturation soils

In the following derivations, we adopt the component forms of tensors for the notations and symbols. For example, in $\dot{A}^{\alpha,\mathrm{n}}_{ijI,j}$, the dot is the time derivative, α represents the value of the α phase, and $\alpha=\mathrm{s},\mathrm{w}$ denotes the solid or water phase, respectively. Omitting α represents the mixture value, superscript n denotes the n-th time step, subscript ij represents the components of tensor A, and "I" denotes the position of node I. If "I" is replaced by "wp" (or "sp"), the value is at the position of water point (or solid point). The comma means the spatial gradient, and the Einstein summation convention is adopted.

Because the dense granular material is assumed to be fully saturated, we employ mixture theory to model the system. The primary assumption is that an arbitrary spatial position possesses two phases, no matter if it is occupied by grains or water. As shown in Fig. 1, the soil with density ρ and porosity n is composed of a water layer with equivalent density $n\rho^w$, which is represented by the set of fluid material points, and a solid layer with equivalent density $(1-n)\rho^s$, which is represented by the set of solid material points. Initially, the two layers of points have the same positions in the same sub-continuum. However, relative motion can occur after deformation. To better understand the mixture theory, the concepts of "class variable" and "spatial position" should be considered. For example, to determine $\dot{v}^s_{iwp}$, the solid velocity class should be built using the solid

Please cite this article as: C. Liu et al., A fully coupled hydro-mechanical material point method for saturated dense granular materials, Powder Technology (2017), http://dx.doi.org/10.1016/j.powtec.2017.02.022