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Numerical study of mixing of binary-sized particles in rotating tumblers on the effects of end-walls and size ratios

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ABSTRACT

The mixing characteristics of binary-sized particles in rotating tumbler are studied by discrete element method (DEM). The cases of diameter ratios of $R_D = 1{:}1$, 2:1 and 3:1 under three rotating velocities $\omega = 1.0, 2.0, 3.0\ \pi$ rad/s with or without friction from front and rear walls of the tumbler are simulated. Short and long tumblers ($L = 0.15$ and 0.3 m respectively) are used with stationary or rotating end-walls. Considering the local granular concentrations of binary-sized particles, an improved information entropy function is proposed to evaluate the overall mixing degree of particles in the tumbler. Moreover, the kinetic energy and the radial distribution function are also explored to discuss the impacts of friction from front and rear walls, the size ratios and the rotating speeds on particle mixing. All the analytical results indicate that the friction from the stationary front and rear walls play a significant role in degrading the flow regime and suppressing particle mixing especially in short tumblers because of the motion resistance. When the effective contact area for end-walls is larger than the lateral wall, it may be even more important than the rotating speed for determining the mixing degree. The lengthened tumbler may attenuate the effect of end-wall frictions. The size ratios play either a positive or a negative role in influencing particle mixing, caused by the static feature of size difference and the dynamical different property inducing size segregation respectively. In addition, the rotations of end-walls may have an effect of mixing enhancement that is reverse to the stationary end-walls.

1. Introduction

Rotating tumblers play significant roles in many industrial applications. The spherical particle mixing phenomena in rotating tumblers have been investigated numerically and experimentally on various aspects. However, the mixing process is still incompletely understood and difficult to predict [1–2].

In particular, the mixing of binary dispersed particles in a three dimensional rotating tumbler is widely applied in engineering industry. For example, Arntz et al. [3] observed the phase behavior of granular beds of bidisperse hard spherical particles in a quasi-two dimensional rotating horizontal drum. The effect of filling level and the rotational velocity of the drum on the mixing and segregation process is investigated. It showed strong correlations between flow regime and segregation pattern. Schutyser et al. [4] predicted the radial and axial mixing characteristics of the monodisperse particles in various 3D slowly rotating drums using discrete particle simulation. Finnie et al. [5] showed the longitudinal and transverse mixing phenomena in horizontal three-dimensional rotary kilns. It was showed that, at identical number of revolutions, the speed of mixing in the transverse plane decreases with increasing rotational speed and filling degree. Savage et al. [6] reported particle size segregation in inclined chute flow. It showed that the small particles falling through the voids in the flowing layer play a positive role in translating the flow regime into a steady state segregation pattern. Reyes et al. [7] studied the pattern developed by a mixture of two types of grains in a triangular rotating tumbler operating in the avalanche regime. There was an invariant zone where the grains do not move relative to the tumbler at the centroid of the triangular tumbler. The explicit expression for the contour of the invariant patterns was obtained. Maione et al. [8] characterized the dynamics of a rotary kiln containing spherical steel balls and wood particles, and investigated the effect of internal baffles and wood chip particle shape on segregation. Soni et al. [9] analyzed mixing of particles in a rotating drum mixer with filling level greater than 50%. It was found that the packing arrangement and particle size significantly affect the formation of the dead zone.

To quantify the mixing degree of particle, many researchers have studied the information entropy. It was firstly proposed by Shannon in 1949 as an entropy-like expression to measure information [10]. The information entropy is defined as the equivalent to uncertainty [11]. With different functions, the information entropy shows different physical interpretations. When the function involves the mixing information of particles, the information entropy could evaluate the degree and

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efficiency of mixing as well as the relationship between parameters of rotating tumbler and mixing degree.

Corresponding to different functions including local concentration, radial distribution functions or coordination number, there are three kinds of information entropy on the study of particle mixing so far. The information entropy based on local concentration could be divided into two opposition and parallel definitions, which rely on the cell and the particle species, respectively. The former definition is widely applied in many granular mixing research [3–5,12–15]. Comparing with the information entropy relying on cell, Gu et al. [16] introduced a parallel definition of the information entropy based on the "per-species entropies" of all types of particles in the mixture. Moreover, Gui et al. [17] presented radial distribution function and corresponding information entropy for the convenience of analysis of mixing. The radial distribution is not only related to the local concentration of particles, but also related to the concentration of particles within a large range of radial distance. Consequently, the information entropy can assess the holistic mixing status of all particles in the prescribed range and be applied in 2D particle mixing. In addition, Ayeni et al. [18] introduced the information entropy based on coordination number, which takes the mixing of particles contacting with each other into account. The information entropy is reliable, especially in the situation that the scale of system is a bit bigger than the scale of particle.

However, the information entropy based on local concentration is not very suitable for quantifying the mixing degree for some specific conditions. For example, when the number of one kind particle is not equal to another, the information entropy may not always increase with the increasing of mixing degree. Consequently, we introduce a modified information entropy function to insure that the mathematical meaning and physical meaning of information entropy could show good coincidences. In addition, the radial distribution function is applied to evaluate the mixing degree in the radial direction.

In general, the ordinary rotating tumblers are composed of the front wall, the rear wall and the lateral wall. The friction on walls plays a significant role for inducing particle motion in rotating tumblers. In previous research, the lateral wall, and front and rear walls of the rotating tumbler were always considered as an entirety. To investigate the effects of friction on walls, especially on the front and rear walls, the binary dispersed particles rotating in the 3D horizontal rotating tumbler with/without friction on front and rear walls are simulated and analyzed. The modified information entropy, radial distribution function and longitudinal local concentration function are used with the aim to analyze the mixing and segregation behavior of particles.

2. Numerical approach

2.1. Discrete element method

The soft-sphere approach of discrete element method is suitable for the particle flow of high concentration where the contact plays a leading role and particle is deformable. Herein, it is used to simulate the motion and collision of particles. In the DEM model [19], the normal force and tangential force model can be governed by:

$$\boldsymbol{F}_n = -k_n\boldsymbol{\xi} - \gamma_n\dot{\boldsymbol{\xi}} \tag{1}$$

$$\boldsymbol{F}_t = -k_t\boldsymbol{\beta} - \gamma_t\dot{\boldsymbol{\beta}} \tag{2}$$

where $\boldsymbol{F}$, k, γ, $\boldsymbol{\xi}$ and $\boldsymbol{\beta}$ are the force, stiffness coefficient, restitution coefficient, inter-particle displacements in normal direction and tangential direction respectively. The superscript 'n', 't' denote normal direction and tangential direction, respectively.

Table 1
Simulation parameters.

Radius of gyration, R_0 (m)	0.8
Length of tumbler, L (m)	0.15; 0.30
Number of particle (N_a, N_b) at $L = 0.15$	(4500, 300), (4500, 3100), (4500, 10,500)
Number of particle (N_a, N_b) at $L = 0.30$	(9000, 600), (9000, 6200), (9000, 21,000)
Particle diameter (D_a, D_b) (mm)	(30, 30), (30, 15), (30, 10)
Particle density, ρ (kg/m^3)	1600
Restitution coefficient, e	0.9
Friction coefficient on wall, γ	0.3
Poisson's ratio of particle, σ_p	0.35
Poisson's ratio of wall, σ_w	0.23
Young modulus of particles, E_p (GPa)	0.2
Young modulus of wall, E_w (GPa)	10
Rotating velocity, ω (π rad/s)	1.0,2.0,3.0
Simulation time step, Δt (s)	1.0×10^{-6}
Total simulation time, T_s (s)	20, 30
Cases[a] for $L = 0.15$ m	$R_D = 1{:}1$, $\omega = 1.0\pi$ rad/s $R_D = 2{:}1$, $\omega = 1.0, 2.0, 3.0$ rad/s $R_D = 3{:}1$, $\omega = 1.0\pi$ rad/s
Cases for $L = 0.30$ m	$R_D = 2{:}1$, $\omega = 1.0, 2.0, 3.0$ rad/s
Cases with rotating end-walls	$R_D = 2{:}1$, $\omega = 1.0, 2.0, 3.0$ rad/s, at $L = 0.15$ m

[a] Notes: 1) $R_D = D_a{:}D_b$; 2) All cases include sub-cases with or without friction on front & back walls.

According to Newton's second law, the governing equations include translational motion and rotational motion of any particle [20]:

$$m_i\frac{d\boldsymbol{v}_i}{dt} = \sum_j \boldsymbol{F}_{c,ij} + \boldsymbol{F}_{g,i} \tag{3}$$

$$\boldsymbol{I}_i\frac{d\boldsymbol{\omega}_i}{dt} = \sum_j \boldsymbol{M}_{ij} \tag{4}$$

where m, $\boldsymbol{v}$, $\boldsymbol{F}_c$, $\boldsymbol{F}_g$, I, $\boldsymbol{\omega}$ and $\boldsymbol{M}$ are the particle mass, the velocity, the contact force, gravity, the moment of inertia, angular velocity and the torque respectively. The simulated parameters are listed in Table 1.

2.2. Numerical setup

The configuration of a cylindrical rotating drum is shown in Fig. 1. Three diameter ratios $R_D = D_A{:}\ D_B = 1{:}1, 2{:}1, 3{:}1$ under three rotating velocities $\omega = 1.0, 2.0, 3.0\ \pi$ rad/s with or without friction on the front and rear walls of the tumbler are simulated. In primary cases, the front and rear walls are stationary either with or without friction. Nevertheless, in the last section (Section 3.4), the effect of rotating end-walls will be compared and discussed.

At first, a number of large spherical particles (type A) are prefilled in the tumbler. Then, the other type of spherical particles (type B, smaller or identical size) will settle down on the top of type-A particles. The particle number is determined by keeping the same or near levels of total particle volumes, i.e. with similar filling levels of the tumbler. After loading type-A and B particles, the tumbler starts to rotate with a predetermined speed. The simulated parameters and cases are listed in Table 1.

2.3. Evaluation function

2.3.1. Improved information entropy for $N_a{:}N_b \neq 1{:}1$

Many researchers adopted information entropy to assess the degree of mixing in the rotating tumbler. The information entropy based on local concentration should have mathematical and physical relations as follow: the greater information entropy should correspond to the better particle mixing degree, or vice versa.