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Credible interval estimation for fraction nonconforming: Analytical and numerical solutions

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A B S T R A C T

This paper proposes a Bayesian statistics-based analytical solution and a Markov Chain Monte Carlo (MCMC) method-based numerical solution to estimate the credible interval for fraction nonconforming. Both solutions provide a more accurate, reliable, and interpretable estimation of sampling uncertainty and can be used to improve the functionality of automated, nonconforming quality management systems. To reveal how the inherent mathematical mechanism functions for an analytical solution, a step-by-step proof with a calculation example is provided. For the numerical solution, a specialized Metropolis-Hastings algorithm and an illustrative simulation example are provided to elaborate the stochastic processes of the method. An industrial case study, from a pipe fabrication company in Alberta, Canada, is presented to demonstrate the feasibility and applicability of the proposed credible interval estimation methods. Results of the case study indicate that both solutions can accurately and reliably serve the nonconforming quality inference purpose. This research can be implemented as a decision-making tool for credible interval estimation and will provide valuable support for understanding and improving quality performance of automated, nonconforming quality control processes.

1. Introduction

Computer-based quality management systems, such as QUALICON or BIM-based quality management systems, have been widely implemented throughout the construction industry [2,8]. Although these systems have facilitated the collection of vast amounts of quality management data, conversion of this data into useable information remains challenging for many practitioners [9]. Automated, data-driven quality management systems, which facilitate the transformation of data into useable information, are often implemented to enhance decision-making processes. However, for a data-driven quality management system to be successful, it must accurately estimate process uncertainty. Integration of accurate, reliable, and straightforward approaches that measure uncertainty of inspection processes are instrumental to the successful implementation of automated, data-driven quality management systems.

Sampling uncertainty must be considered during estimation of a true population variable when data are obtained from a sample rather than an entire population [22]. A common tool used to assess uncertainty is interval estimations, which are applied to estimate the margin of sampling error [7]. Of the several types of interval estimations, confidence intervals, which are commonly introduced in statistics textbooks, have been widely applied in statistical process control. However, several researchers have outlined the disadvantages of confidence intervals and have contended that confidence intervals are not well-suited to address the needs of scientific research [16]. Accordingly, due to their straightforwardness [7] and reliability [10], researchers are now advocating for the use of Bayesian credible intervals rather than conventional confidence intervals. In contrast to confidence intervals, an observer can combine previous knowledge with observed data to estimate parameters of interest when using Bayesian statistics [3,10]. In a Bayesian treatment, prior distributions of the parameters are introduced and posterior distributions are computed, based on Bayes' theorem, from observed data [4]. After obtaining posterior distributions, uncertainty can be quantified by providing certain tail quantiles of the posterior distribution [21]. For example, a 95% credible interval can be specified by the 0.025 and 0.975 quantiles of the posterior distribution.

Calculation of sampling uncertainty in quality management systems is further complicated for quality characteristics that cannot be appropriately represented numerically. Often, quality characteristics are assessed as either conforming or nonconforming to specified quality standards. In contrast to data that is represented numerically, sampling uncertainty must instead be assessed from the fraction nonconforming, defined as the ratio of nonconforming items in a population to the total items in that population [15]. To appropriately incorporate

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uncertainty, it is necessary to obtain a range of values that cover the true population fraction nonconforming [17]. As is common for statistical processes, this range should be wider for unfamiliar items and narrower for familiar items.

The aim of the present study is to introduce a credible interval estimation approach for fraction nonconforming by providing two alternative types of solutions, namely analytical and numerical, to more effectively incorporate uncertainty in fraction nonconforming inferences. The content of this paper is organized as follows: An overview of the research workflow is provided in the methodology section, and the research methodology is detailed in the following sections. First, the statistical principles underlying fraction nonconforming for mathematically modelling nonconforming quality control processes are discussed. Then, a detailed introduction to credible interval and Bayesian inference is provided. Afterwards, a Bayesian statistics-based analytical solution and an MCMC method-based numerical solution for determining credible intervals and posterior distributions for fraction nonconforming are introduced. To elaborate on the implementation of the proposed solutions, an illustrative example of each solution is provided. Finally, the feasibility, applicability, and consistency of the two proposed solution types are demonstrated following a practical case study of industrial pipe welding quality management. In addition to providing insights for the improvement of uncertainty estimation in automated data-driven quality management systems, findings of this study will also provide valuable insights on the use of Markov Chain Monte Carlo (MCMC) methods to determine posterior distributions for complex variables.

2. Research methodology

The research methodology of this study is illustrated in Fig. 1. First, the problem was abstracted into a mathematical model using a Bernoulli process—an established model from the area of statistical quality control—to estimate the fraction nonconforming [15]. Second, to demonstrate the advantages of implementing Bayesian statistics for incorporating uncertainty in fraction nonconforming estimation, the theoretical background of credible interval estimation and Bayesian inference were thoroughly investigated. From this, it was determined that a credible interval has a more intuitive interpretation than a classic confidence interval when estimating the unknown fraction nonconforming. The results also demonstrated that Bayesian statistics was capable of recalibrating existing statistical distributions with newly updated data. To determine a non-informative prior distribution for fraction nonconforming estimation, selection of the prior distribution was then investigated. Finally, a Bayesian statistics-based analytical solution and an MCMC-based numerical solution were developed to derive the posterior distribution of fraction nonconforming. To reveal how the inherent mathematical mechanism functions, a step-by-step proof with a calculation example was conducted for the analytical solution; a specialized Metropolis-Hastings algorithm and an illustrative simulation example were provided for the numerical solution. Advantages and disadvantages of each method were discussed. Then, the feasibility and applicability of the proposed solutions were evaluated following their application to an industrial case study. Details of the systematic and theoretical analysis of these research steps are detailed as follows.

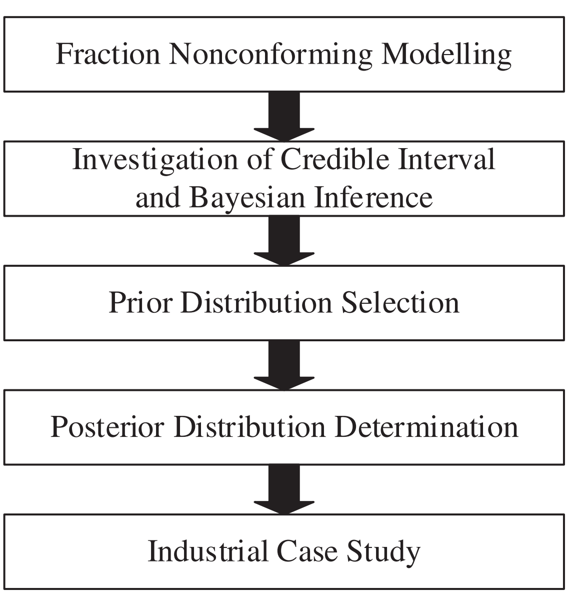

Fig. 1. Research methodology flow chart.

3. Fraction nonconforming modelling

In the nonconforming quality inspection process, the desired outcome is usually referred to as "success" and the alternative outcome is often referred to as "failure." When an item fails, it must be repaired and inspected until it passes inspection. The inspection outcome O can be treated as a Bernoulli random variable with probability function [15]:

$$P(O) = \begin{cases} p & x = 1 \\ (1-p) = q & x = 0 \end{cases} \tag{1}$$

Variable O takes on a value of 1 with probability p and the value 0 with probability $(1-p) = q$. A realization of this random variable is called a Bernoulli trial. The sequence of Bernoulli trials is a Bernoulli process. The number of failed inspections X has a binomial distribution $B(n,p)$.

In statistical quality control processes, the fraction nonconforming of the sample is defined as the ratio of the number X of nonconforming items in the sample to the sample size n as Eq. (2) [15].

$$\hat{p} = \frac{X}{n} \tag{2}$$

$\hat{p}$ is a point estimate of the true, unknown value of the binomial variable p, which represents the fraction nonconforming of the sampled items. The mean of $\hat{p}$ can be calculated as Eq. (3).

$$\mu_{\hat{p}} = p \tag{3}$$

4. Credible interval and Bayesian inference

In statistics, interval estimation is generally defined as the use of sample data to calculate an interval of possible (or probable) values of an unknown population variable [7]. Confidence intervals and credible intervals are the most widespread forms of interval estimations. In general, both confidence intervals and credible intervals can be defined for a variable X as $P\{l \le X \le u\} = 100(1-\alpha)\%$. Where l is the lower interval limit, u is the upper interval limit, and $(1-\alpha)$ is the level of confidence (α is the significance level). However, the interpretation for confidence intervals and credible intervals is conceptually different.

Before introducing the concept of the credible interval, the drawbacks of the confidence interval will be discussed. Generally, a confidence interval is a range of values designed to include the true value of the variable with a tolerance probability of $100(1-\alpha)\%$. As the number of failed inspections has a binomial distribution, only confidence intervals for binomial distributions will be discussed here. The Wald's interval, Wilson interval, and Agresti-Coull interval are classical methods for setting confidence intervals for binomial distributions [6]. Their analytical equations are listed in Table 1.

From the confidence interval equations listed in Table 1, it is evident that interval endpoints in these intervals depend only on collected data (i.e., the fraction nonconforming $\hat{p}$ and the sample size n).